

Section 2.6 Linear Inequalities

Objectives

In this section, you will learn to:

- Interpret words of comparison.
- Compare two numbers using a number line.
- Graph inequalities on a number line.
- Switch sides of an inequality
- Check for possible solutions of an inequality.

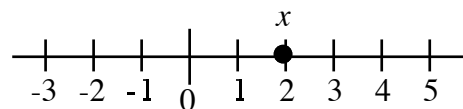
To successfully complete this section, you need to understand:

- The number line (1.2)
- Comparing integers (1.2)

INTRODUCTION

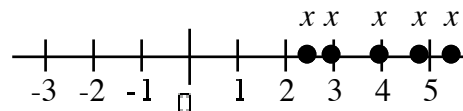
Recall that an equation is a mathematical sentence in which two expressions are equal in value, and we use an equal sign ($=$) to express this equality, such as $2x + 3 = x + 5$. We have also learned that this type of equation, called a *linear* equation, has just one solution, one number that makes it true: $x = 2$.

This single solution could be represented on the number line by locating the variable, x , at 2, represented by a single dot:

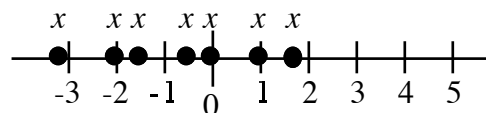


In this section we look at mathematical sentences that have many solutions. These statements do not express equality. Instead, they express an *inequality*, indicating that one value is different from another value in some way.

For example, numbers greater than 2 are to the right of 2 on the number line, and they are not equal to 2.



Likewise, numbers less than 2 are to the left of 2 on the number line, and they are not equal to 2.



In this section, we look at understanding inequalities and how to graph them. Then, in Section 2.7, we learn how to solve them, similar to how we solved equations earlier in this chapter.

WORDS OF COMPARISON

When two expressions are not equal to each other, the statement is called an **inequality**. Inequalities come in three basic forms:

Words of comparison	Symbol	Example
is not equal to	\neq	Groucho Marx's age <u>is not equal to</u> Harpo Marx's age: Groucho's age \neq Harpo's age
is greater than	$>$	The population of Ohio <u>is greater than</u> the population of Vermont: population of Ohio $>$ population of Vermont
is less than	$<$	The price of a Smart Car <u>is less than</u> the price of a BMW: price of a Smart Car $<$ price of a BMW

In the examples, above, we can be assured that each is true. Sometimes, though, we are given a sentence of comparison that is not always true. For example, "The number of boys is less than the number of girls," may or may not be true. It all depends on the group of children to which we are referring.

<u>Think about it 1</u>	For each, write an example that illustrates the given inequality.
a) \neq	_____
b) $>$	_____
c) $<$	_____

In algebra, it is appropriate to let a variable represent an unknown number. For example, in the sentence, "The number of children is less than 10," we can let $x =$ the number of children and then represent the sentence in algebra as

$$x < 10.$$

This means that x (the number of children) can be 9 or fewer. In fact, according to the sentence, it's even possible that there are 0 children (no children at all). But, does the sentence allow for the possibility of there being 10 children? No, not the way the sentence is written.

What if we wanted to include the possibility of 10 children? We might say that there could be 10 or fewer children. In mathematics, we could say, "The number of children is less than or equal to 10." This kind of sentence allows for two additional forms of inequalities:

- d) “**is greater than or equal to**” (\geq) as in, “The price of a gallon of orange juice is greater than or equal to the price of a gallon of milk” or

the price of orange juice \geq the price of milk

- e) “**is less than or equal to**” (\leq) as in, “The number of minutes watching a movie in the theater is less than or equal to the number of minutes watching the movie on television” or

time watching a movie in the theater \leq time watching a movie on television.

Think about it 2 For each, write an example that illustrates the given inequality.

a) \geq _____

b) \leq _____

There are special phrases that mean one inequality or another. To translate English into algebra, we must think about the meaning of each.

Here’s the situation: you are preparing for a birthday party for some children. Think of each as the number of chairs required in the situation.

For each, **Let x = the number of children going to the party.**

Phrase	Meaning	Example	Algebra
fewer than	less than	There are fewer than 10 children. (9 chairs would be enough, maybe even fewer than that.)	$x < 10$
more than	greater than	There are more than 10 children. (11 chairs is the minimum needed; 10 chairs would not be enough.)	$x > 10$
at least	greater than or equal to	There are at least 10 children. (10 is the fewest number of chairs needed, but you may need <i>more</i> than that.)	$x \geq 10$
at most	less than or equal to	There are at most 10 children. (10 is the most chairs needed, but it could be that you need fewer than that.)	$x \leq 10$
no fewer than	greater than or equal to	There are no fewer than 10 children. (You can’t have fewer than 10 chairs; you must have <i>at least</i> 10—10 or more—chairs.)	$x \geq 10$
no more than	less than or equal to	There are no more than 10 children. (There’s not going to be 11 children; you might need as many as 10 chairs, but possibly less.)	$x \leq 10$

Example 1: Translate each of these sentences into algebra. For each, Let $x =$ the number of games.

- a) Nikki's team will play more than 8 games this season: $x > 8$
- b) Juan's team will play at least 12 games this season. $x \geq 12$
- c) Lonny's team will play less than 10 games: $x < 10$
- d) Shari's team will play no more than 9 games: $x \leq 9$
- e) Kyle's team will play at most 15 games: $x \leq 15$

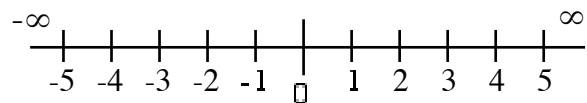
YTI 1

Translate each of these sentences into algebra. For each, Let $x =$ the number of children. Use Example 1 as a guide.

- a) Nate invited less than 15 children to his birthday party. _____
- b) Sarah has no more than 23 children in her class. _____
- c) There were at least 20 children in the play. _____
- d) The basketball league will have at most 8 children on a team. _____
- e) Mr. Simpson has said that more than 5 children will get an A. _____
- f) Fewer than 8 children signed up to go on the field trip. _____

VISUALIZING INEQUALITIES

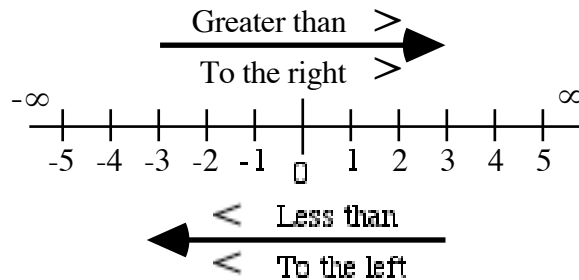
We can use the number line to help us visualize inequalities. Though this number line shows the location of a few integers, we know that it extends indefinitely to the right and to the left.



We use the symbol of a *lazy eight*, ∞ , to represent **infinity**, the idea that there is no largest positive number. Likewise, $-\infty$ tells us that there is no largest negative number.

When we think of numbers along the number line, *less than* means *to the left of* and *greater than* means *to the right of*.

Notice that the less than sign, $<$, points left and the greater than sign, $>$, points right. In this way, we can talk about the *direction* of the inequality sign.



Caution: When determining whether one number is greater than or less than another number, it is always best to refer to the number line. For example, negative numbers that have a large numerical value, such as -20 , are actually less than smaller negative numbers, such as -5 : $-20 < -5$.

Think about it 3 Use the words *less than* or *greater than* to fill in the blank. Explain your answer.

Every positive number is _____ every negative number.

Explain your answer _____

Think about it 4 Use the words *less than* or *greater than* to fill in the blank. Explain your answer.

Every negative number is _____ every positive number.

Explain your answer _____

Example 2: Fill in the box with an inequality sign (either $<$ or $>$) that makes the statement true.

a) $2 \square 9$ b) $8.3 \square 6.25$ c) $\frac{3}{4} \square -5$

d) $-\frac{2}{3} \square \frac{7}{10}$ e) $-30 \square -1.8$ f) $-4.3 \square -9$

Procedure: Use a number line as a guide and the idea that *less than* means *to the left of* and that *greater than* means *to the right of*.

Answer: a) $2 \square < \square 9$ b) $8.3 \square > \square 6.25$ c) $\frac{3}{4} \square > \square -5$

2 is to the left of 9

8.3 is to the right of 6.25

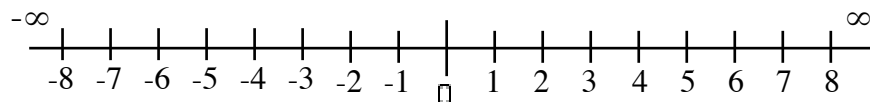
$\frac{3}{4}$ is to the right of -5

d) $-\frac{2}{3} \square < \square \frac{7}{10}$ e) $-30 \square < \square -1.8$ f) $-4.3 \square > \square -9$

$-\frac{2}{3}$ is to the left of $\frac{7}{10}$

-30 is to the left of -1.8

-4.3 is to the right of -9



YTI 2

Fill in the box with an inequality sign (either $<$ or $>$) that makes the statement true. (You may use the number line, above, to help you think about the answers.) Use Example 2 as a guide.

a) $8.9 \square 3$ b) $2.5 \square -10$ c) $-6.7 \square 2.4$

d) $-\frac{4}{9} \square \frac{2}{5}$ e) $-14 \square -3.8$ f) $-\frac{3}{8} \square 0$

g) $-2.9 \square -7.3$ h) $0.3 \square -1$ i) $0 \square -5.5$

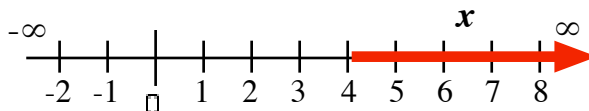
GRAPHING ON THE NUMBER LINE

When an inequality contains a variable, such as $x > 4$, then the variable, x , represents all real numbers that make the inequality true. The collection of all of the solutions of an inequality is called the **solution set**.

For $x > 4$, the solution set is all real numbers to the right of 4, not just the integers.

For example, if the price of a hat is more than \$4.00, such as $h > 4.00$, we are not restricted to only dollar amounts, such as \$5.00 or \$6.00. We must consider prices that contain pennies as well, such as \$4.01, \$4.26, and \$5.33. In other words, we must consider all real numbers.

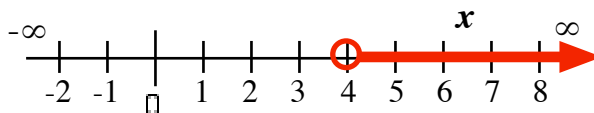
To represent the solution set on the number line, we use a thick line extending to the right of 4.



The arrow on this thick line indicates that we consider all values to the right of 4, not just the ones we can see on the number line.

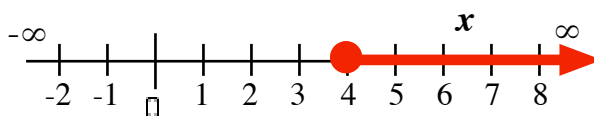
There is an x above the thick line to indicate that every number the thick line covers is in the solution set for x .

To indicate that 4 is not a possible value of x , we use an open circle at 4. This means that 4 is not in the solution set.



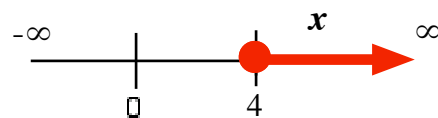
How would the graph look if we wanted to include 4 in the solution set for x ?

First, the inequality would have to allow for the variable to be equal to 4. This is a different inequality, namely $x \geq 4$.



Second, to show that 4 is to be included, we use a closed circle at 4.

To be able to graph inequalities more efficiently, we can abbreviate the number line and include only the most important features, namely the origin (0) and the starting number in the inequality.



Abbreviating the number line this way allows us to represent inequalities with larger starting values.

The graphs for the other two inequality signs, $<$ and \leq , graph in a similar fashion.

Example 3: Draw the graph of the solution set for each variable.

a) $y < 2$

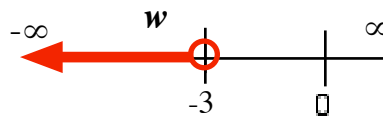
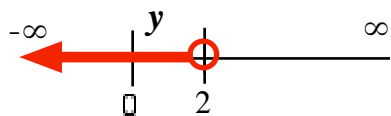
b) $w \leq -3$

c) $x > -15$

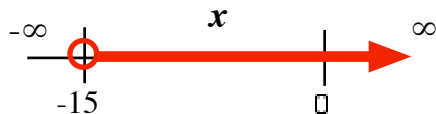
Answer: Be sure to include the variable and the infinities.

a) $y < 2$

b) $w \leq -3$



c) $x > -15$



YTI 3

Draw the graph of the solution set for each variable on the number line provided. Use Example 3 as a guide.

a) $m \geq -7$

b) $v < 2$

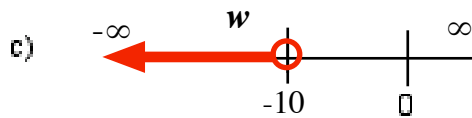
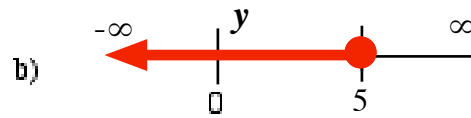
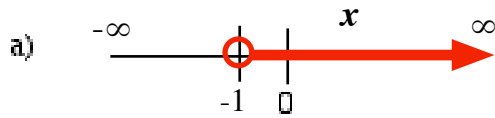
c) $x \leq -3$

d) $y > 9$

e) $p < 0$

f) $w \geq 0$

Example 4: Write an inequality based on the given graph..

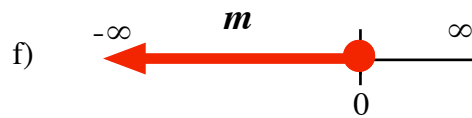
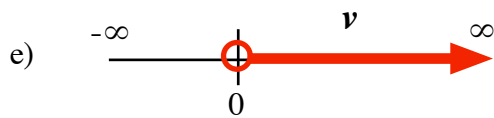
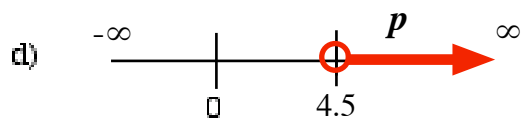
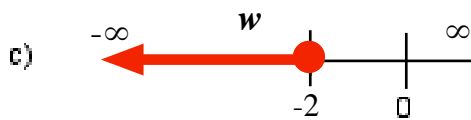
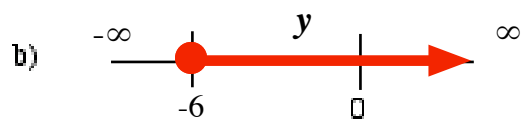
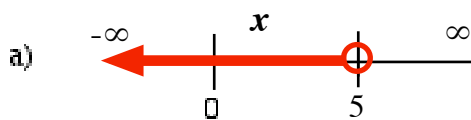


Procedure: First, be sure to notice the variable above the graph. Second, identify whether the circle is open (no equal sign in the inequality) or closed (include the equal sign in the inequality), and third note the direction of the arrow. It should match the direction of the inequality sign.

- Answer:**
- a) $x > -1$ Open circle means no equal sign; points right means greater than, $>$.
 - b) $y \leq 5$ Closed circle means include the equal sign; points left means less than, $<$.
 - c) $w < -10$ Open circle means no equal sign; points left means less than, $<$.
 - d) $v \geq 3.5$ Closed circle means include the equal sign; points right means greater than, $>$.

YTI 4

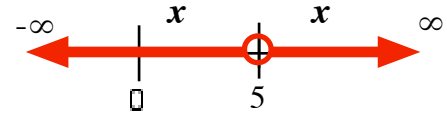
Write an inequality based on the given graph. Use Example 4 as a guide.



THE NOT EQUAL SIGN

The not equal sign, \neq , is used to say that one value does not equal another. To say that $a \neq b$ means that a could be less than b or a could be greater than b ; it just can't be equal to b .

For example, $x \neq 5$ means that x can be any real number except for 5. We represent this on the number line with a thick line covering the whole number line except for an open circle at 5.



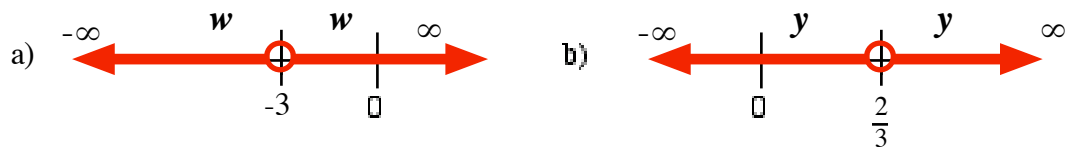
Notice that in the graph, the variable, x , is written above both on the less than side and on the greater than side of 5. This is to indicate that all of the are possible solutions for x .

Example 5: Draw the graph of the solution set for each variable.

a) $w \neq -3$ b) $y \neq \frac{2}{3}$

Procedure: Be sure to include the variable and the infinities.

Answer:



YTI 5

Draw the graph of the solution set for each variable on the number line provided. Use Example 5 as a guide.

a) $m \neq 4$

b) $v \neq -2$

c) $x \neq -\frac{5}{4}$

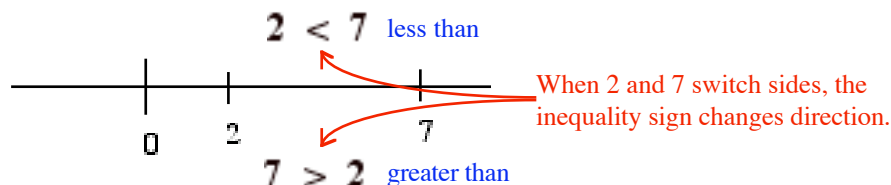
d) $p \neq 0$

SWITCHING SIDES OF AN INEQUALITY

As has been mentioned, the direction of the inequality sign, $<$ or $>$, can be helpful in determining the direction to graph on the number line. The less than sign *points* to the left and the greater than sign *points* to the right.

First notice that both of these statements are true:

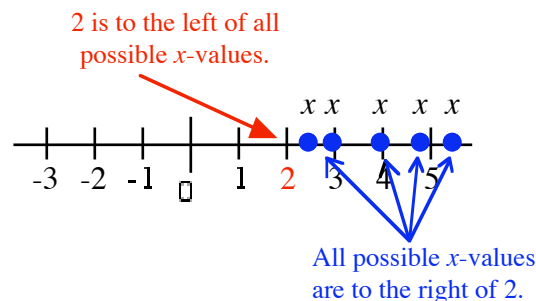
- (1) 2 is less than 7: $2 < 7$ } Note that when the numbers “switch sides,”
 (2) 7 is greater than 2: $7 > 2$ } the inequality sign changes *direction* from
 less than to greater than.



If an inequality contains a variable, then the ideas that $<$ means *pointing to the left* and $>$ means *pointing to the right* is valid only when the variable is on the left side.

What if, instead of 7, we see $2 < x$?

On the number line, this means that 2 is to the left of all possible x values. It also means that the possible x values are all to the right of 2.



$$2 < x$$

$$x > 2$$

Notice that as the 2 and x switch sides, the direction of the inequality sign changes from less than to greater than. We get an equivalent statement and one that is easier to graph: $x > 2$.

It is important to notice this because the idea that *the direction of the inequality sign indicates the direction of the graph* is true only if the variable is on the left side. So, if the variable is on the right side, we must prepare to graph by first switching sides and placing the variable on the left.

Example 6: Write an equivalent statement for the inequality by switching sides. Graph the resulting solution set.

a) $3 > y$

b) $5 \leq w$

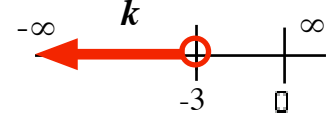
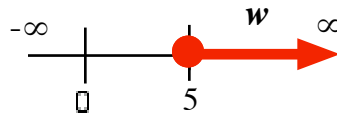
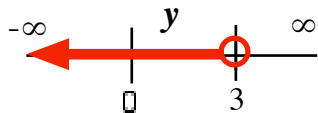
c) $-3 \geq k$

Procedure: As we switch sides, we must also change the direction of the inequality sign.

a) $y < 3$

b) $w \geq 5$

c) $k \leq -3$



YTI 6

Write an equivalent statement for the inequality by switching sides. Graph the resulting solution set. Use Example 6 as a guide.

a) $4 > w$

b) $6 \leq x$

c) $-3 \geq p$

d) $-8 < c$

e) $0 > n$

f) $0 \leq b$

SOLVING INEQUALITY STATEMENTS: THE SOLUTIONS

In algebra, to solve means to find the solution. In a linear equation, the solution is the single number that makes the equation true. For example, in $2x + 7 = 1$, the solution is $x = -3$. This is the only number that will make the statement (the equation) true. This is demonstrated by putting the value -3 in for x in the equation:

$$\begin{aligned}
 x = -3: \quad 2x + 7 &= 1 \\
 2(-3) + 7 &= 1 \\
 -6 + 7 &= 1 \\
 1 &= 1 \quad \checkmark \text{ True, so the solution is } -3.
 \end{aligned}$$

In an inequality, a **solution** is still any number that makes the statement true.” However, there are many more solutions than just one. In fact, there are an infinite number of solutions, too numerous to mention, so we use an inequality statement, such as $x > -3$.

For example, to determine whether a particular number, such as 8 or -2, is a solution for $5x - 7 > 3x - 3$, we can first replace x with 8 in the statement, and then replace x with -2 in the statement, as shown here:

	$5x - 7 > 3x - 3$		$5x - 7 > 3x - 3$
$x = 8$:	$5(8) - 7 > 3(8) - 3$	$x = -2$:	$5(-2) - 7 > 3(-2) - 3$
	$40 - 7 > 24 - 3$		$-10 - 7 > -6 - 3$
	$33 > 21$		$-17 > -9$
	True		False

So, 8 is a solution but -2 is not. There are many other numbers that are solutions and many others that are not.

Example 7: Decide whether the given value of x is a solution of $3 - 2x \leq 3x + 23$.

a) $x = 2$

b) $x = -6$

c) $x = -4$

Procedure: Substitute the value for each x in the inequality statement. If the value makes a true statement, then it is a solution. If it makes a false statement, then it is not a solution.

Answer: a) **$x = 2$:** $3 - 2(2) \leq 3(2) + 23$
 $3 - 4 \leq 6 + 23$
 $-1 \leq 29$ True. 2 is a solution.

b) **$x = -6$:** $3 - 2(-6) \leq 3(-6) + 23$
 $3 + 12 \leq -18 + 23$
 $15 \leq 5$ False. -6 is not a solution.

c) **$x = -4$:** $3 - 2(-4) \leq 3(-4) + 23$
 $3 + 8 \leq -12 + 23$
 $11 \leq 11$ True. -4 is a solution.

Note: In Example 6c), we find that -4 is a solution because $11 = 11$. It is true that 11 is not less than 11, but the equality option in the symbol \leq allows for the possibility that the left side is equal to the right side.

Contrast this with the results in Example 8:

Example 8: Decide whether 7 is a solution of $3w + 5 > 4w - 2$.

Answer: $w = 7$: $3(7) + 5 > 4(7) - 2$
 $21 + 5 > 28 - 2$
 $26 > 26$ False. 7 is not a solution.

Here, 26 is not *greater* than itself, and there is not an option for it to be equal, therefore, $w = 7$ makes the inequality statement false, and 7 is not a solution.

YTI 7

Decide whether the given value of x is a solution of $3x - 6 < 9 - 2x$. Use Examples 7 and 8 as guides.

a) $x = 6$

b) $x = -2$

c) $x = 0$

d) $x = 4$

e) $x = -1$

f) $x = 3$

For the inequality statement $3x - 6 < 9 - 2x$, it turns out that every number less than 3 is a solution. We can write this as $x < 3$. In Section 2.7 we will learn how to find the set of solutions by *solving* the inequality, using step-by-step algebra.

Answers: You Try It and Think About It

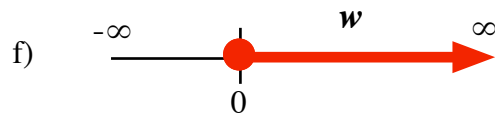
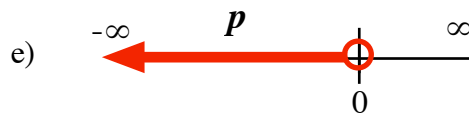
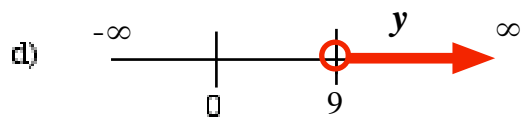
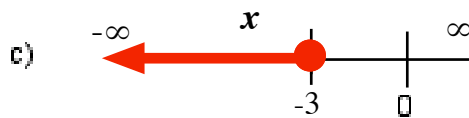
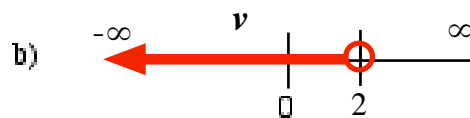
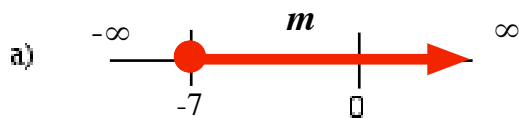
YTI 1:

- a) $x < 15$ b) $x \leq 23$ c) $x \geq 20$ d) $x \leq 8$
 e) $x > 5$ f) $x < 8$

YTI 2:

- a) $8.9 > 3$ b) $2.5 > -10$ c) $-6.7 < 2.4$ d) $\frac{4}{-9} < \frac{2}{5}$
 e) $-14 < -3.8$ f) $\frac{3}{-8} < 0$ g) $-2.9 > -7.3$ h) $0.3 > -1$
 i) $0 > -5.5$

YTI 3:



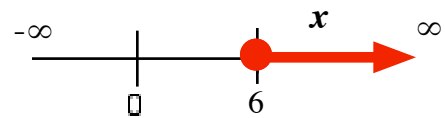
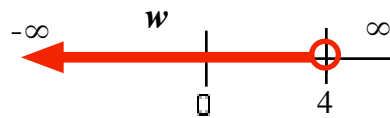
YTI 4:

- a) $x < 5$ b) $y \geq -6$ c) $w \leq -2$ d) $p > 4.5$
 e) $v > 0$ f) $m \leq 0$

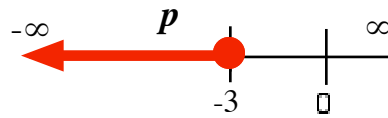
YTI 5:

a) $w < 4$

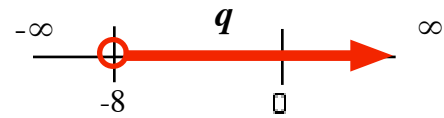
b) $x \geq 6$



c) $p \leq -3$



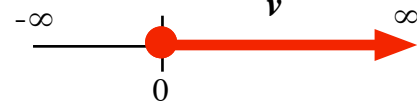
d) $q > -8$



e) $n < 0$

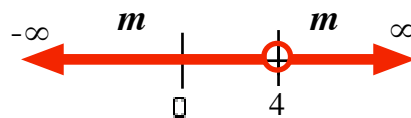


f) $v \geq 0$

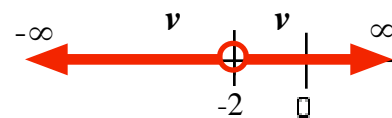


YTI 6:

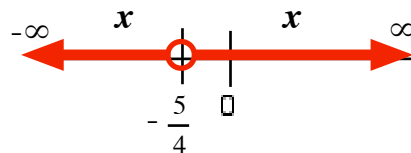
a) $m \neq 4$



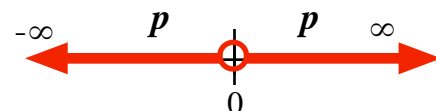
b) $v \neq -2$



b) $x \neq -\frac{5}{4}$



d) $p \neq 0$



YTI 7:

- a) $12 < -3$; False; 6 is not a solution. b) $-12 < 13$; True; -2 is a solution.
 c) $-6 < 9$; True; 0 is a solution. d) $6 < 1$; False; 4 is not a solution.
 e) $-9 < 11$; True; -1 is a solution. f) $3 < 3$; False; 3 is not a solution.

Think About It:

1. Answers may vary. 2. Answers may vary. 3. greater than; Every positive number is to the right of every negative number, so every positive number must be greater than every negative number. 4. less than; Every negative number is to the left of every positive number, so every negative number must be less than every positive number.

Section 2.6 Exercises

Think Again.

1. Is it possible for $3 \leq 3$? Explain your answer.
2. Is it possible for $-4 \geq -4$? Explain your answer.
3. If a is a positive number then is $-a$ less than or greater than a ? Explain your answer.
4. If a is a negative number then is $-a$ less than or greater than a ? Explain your answer.

Focus Exercises.

Translate each of these sentences into algebra. For each, Let $x =$ the number of cars.

- | | |
|---|---|
| 5. Janet has owned more than 12 cars in her lifetime. | 6. There were at most 6 cars parked in front of Joe's house. |
| 7. There were no fewer than 30 cars in the parking lot. | 8. There were fewer than 10 cars following the wedding couple's limo. |

Translate each of these sentences into algebra. For each, Let $x =$ the number of golf balls.

- | | |
|---|---|
| 9. Tamara prefers to hit no more than 30 golf balls on the driving range. | 10. Rey always loses at least 5 golf balls whenever he plays golf. |
| 11. Carla carries fewer than 12 golf balls at a time in her golf bag. | 12. Arnie usually uses more than 4 golf balls at a time on the putting green. |

Translate each of these sentences into algebra. For each, Let $x =$ the number of students.

- | | |
|---|--|
| 13. There were at most 8 students who got an A on the test. | 14. There were no fewer than 20 students waiting in line at the bookstore. |
| 15. At least 15 students wanted to add the class on Monday. | 16. No more than 12 students attended the Art Club's meeting last week. |

Fill in the box with an inequality sign (either $<$ or $>$) that makes the statement true. (You may use the number line, above, to help you think about the answers.)

17. $-9 \square 4$

18. $-8 \square -9$

19. $-7 \square 3$

20. $5 \square -1$

21. $-2.5 \square -4.1$

22. $-3.1 \square -3.9$

23. $\frac{3}{4} \square -7$

24. $-\frac{5}{6} \square -1\frac{3}{10}$

25. $0 \square -6$

26. $0 \square \frac{3}{8}$

27. $0.08 \square 0$

28. $-\frac{1}{6} \square 0$

Draw the graph of the solution set for each variable on a number line. Be sure to include the variable and the infinities, along with the origin and the graph.

29. $x < 4$

30. $y < -1$

31. $p > -3$

32. $w > 7$

33. $v \geq -3$

34. $n \geq 6$

35. $x \leq 5$

36. $k \leq -4$

37. $u > 0$

38. $r < 0$

39. $q \leq 0$

40. $v \geq 0$

41. $k \leq \frac{3}{8}$

42. $m < -\frac{4}{5}$

43. $w > -1\frac{2}{3}$

44. $y \geq 2\frac{5}{6}$

45. $x > 4.3$

46. $y < 2.6$

47. $p \leq -1.8$

48. $w \geq -5.6$

49. $u \neq -6$

50. $q \neq -1$

51. $n \neq 2$

52. $v \neq 5$

53. $y \neq 0$

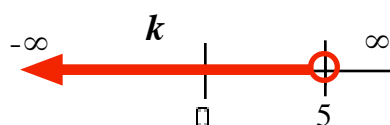
54. $p \neq -\frac{1}{2}$

55. $w \neq 3\frac{1}{4}$

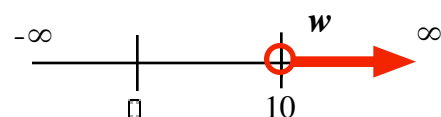
56. $x \neq -2.5$

Write an inequality based on the given graph.

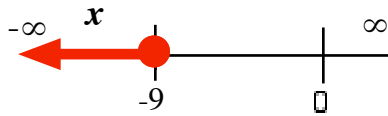
57.



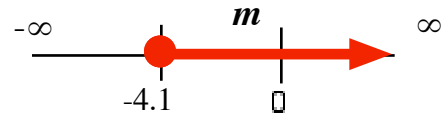
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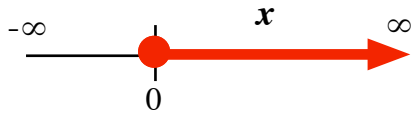
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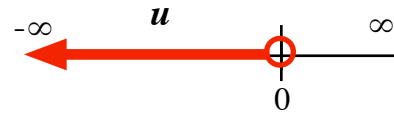
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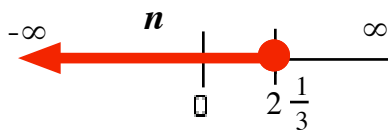
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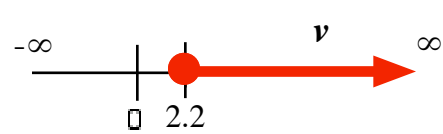
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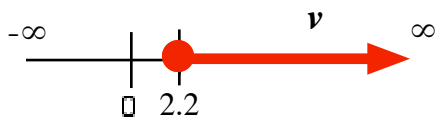
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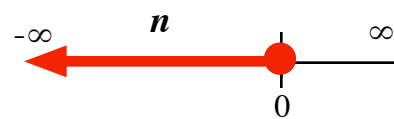
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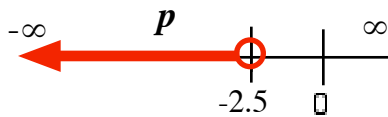
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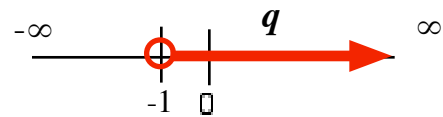
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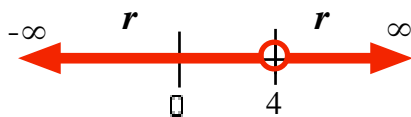
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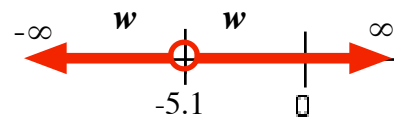
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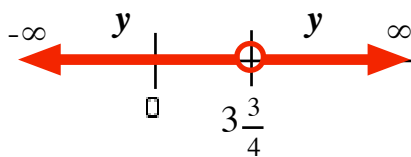
69.



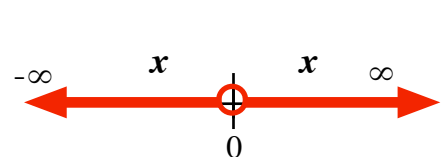
70.



71.



72.



Write an equivalent statement for the inequality by switching sides. Graph the resulting solution set.

73. $-6 \geq y$

74. $-1 \leq r$

75. $0 < w$

76. $0 > h$

77. $4 < n$

78. $5 \geq x$

79. $2.5 \leq u$

80. $-3.2 < v$

81. $4\frac{1}{5} \geq x$

82. $2\frac{1}{6} \leq p$

83. $-\frac{3}{5} > m$

84. $-\frac{4}{9} < y$

Decide whether the given value of y is a solution of $3y + 2 < y - 4$.

85. $y = 3$

86. $y = -3$

87. $y = 0$

88. $y = -5$

Decide whether the given value of x is a solution of $4 - x \geq 3x - 8$.

89. $x = 6$

90. $x = 3$

91. $x = 0$

92. $x = -2$

Decide whether the given value of w is a solution of $w - 6 \geq \frac{1}{2}w + 3$.

93. $w = -4$

94. $w = 6$

95. $w = 10$

96. $w = 0$

Think Outside the Box:

Follow these steps for each given inequality.

- Replace x with 2 in the given inequality. Is 2 a solution?
- Multiply each and every term in the inequality by -1 to create a new inequality.
- In the new inequality, replace x with 2. Is 2 a solution?
- If 2 is not a solution, how can the new inequality be changed to make 2 a solution?

97. Given inequality: $x - 5 < -1$.

98. Given inequality: $3x - 6 > -4$.

99. Given inequality: $x - 2 < 2x + 3$.

100. Given inequality: $\frac{1}{2}x - 5 > -4$.

101. Based on your results in each of these four exercises, write a conclusion about multiplying the terms of an inequality by -1.