Section 4.2 Graphing Lines

Objectives
In this section, you will learn to:

- Identify collinear points.
- Graph the line of an equation of the form $ax + by = c$.
- Graph the line of an equation of the form $y = mx + b$.
- Graph a line using a different scale.

To successfully complete this section, you need to understand:

- The order of operations (1.6)
- Placing values into formulas (1.8)
- Plotting points in the *x*-*y*-plane (4.1)
- Drawing a line in the *x*-*y*-plane (4.1)

INTRODUCTION

A **linear equation** of two variables, *x* and *y*, is one in which the ordered pair solutions are points on a line in the *x*-*y*-plane. Because the ordered pairs are both solutions and points, there will be times in this text when they are both referred to as *points*.

In Section 4.1 we saw a few linear equations, including $x - y = 4$, $y = \frac{1}{2}x + 3$, and $3x + y = -2$. Each of these equations was graphed as a line in the *x*-*y*-plane, and even though the fewest number of points needed to draw a line is two, we usually plot at least three points to help us draw the line more accurately.

COLLINEAR POINTS

Here, again, are some of the solutions to the linear equation $x - y = 4$. There are an infinite number of solutions, each an ordered pair, and those ordered pairs can be plotted in the *x*-*y*-plane.

In table form

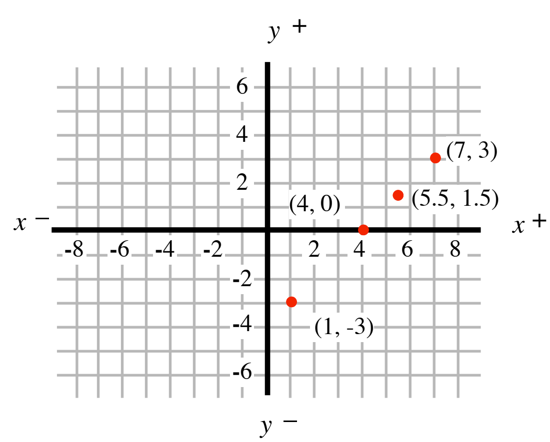
x	*y*
7	3
5.5	1.5
4	0
1	-3

As ordered pairs

(*x*, *y*)
(7, 3)
(5.5, 1.5)
(4, 0)
(1, -3)

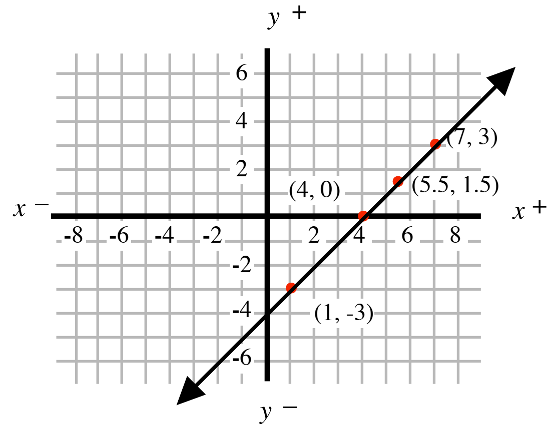
and many more....

As points in the *x*-*y*-plane



Two or more points that form a line—or are already on the same line—are said to be **collinear** (co-lih´nee-er).

Because these ordered pairs came from the same linear equation, $x - y = 4$, the corresponding points in the *x*-*y*-plane are collinear and we can draw a (straight) line through them.



As the line extends through the *x*-*y*-plane, we can see other points on the line, such as (0, -4) and (-2, -6).

Do we need all of these points to <u>draw</u> the line? *No*. In fact, as stated in Section 4.1, we can use as few as two points, because *any two points determine a unique line*.

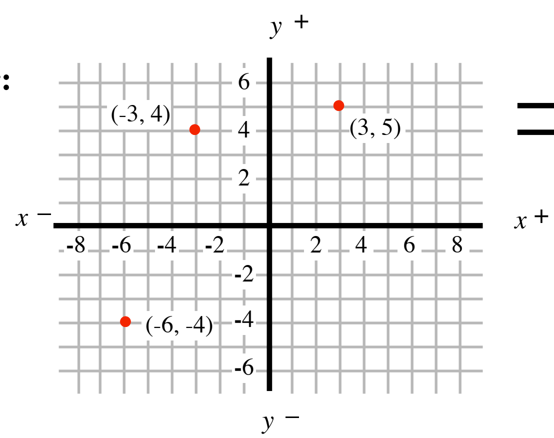
Do any *three* points determine a unique line? *No*. Three points determine a unique line only if the points are *collinear*. In fact, if three points are not collinear, then they actually determine *three* lines, as demonstrated in Example 1.

Example 1: For this set of three non-collinear points, draw the three lines that pass through each pair.

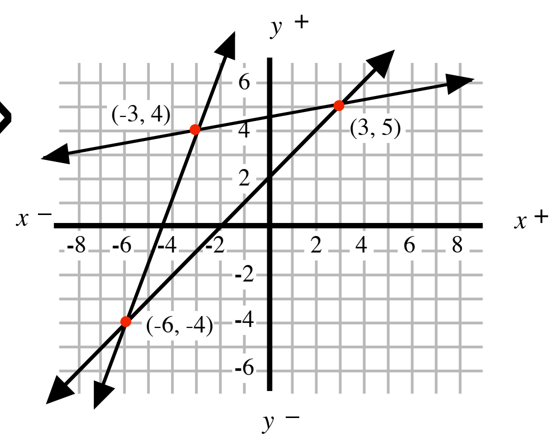
(3, 5), (-3, 4) and (-6, -4).

Procedure: Plot the points and draw the three lines connecting two of them at a time.

Answer:

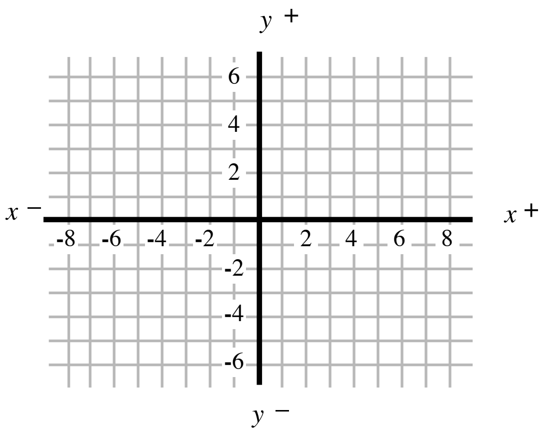


$\Rightarrow$

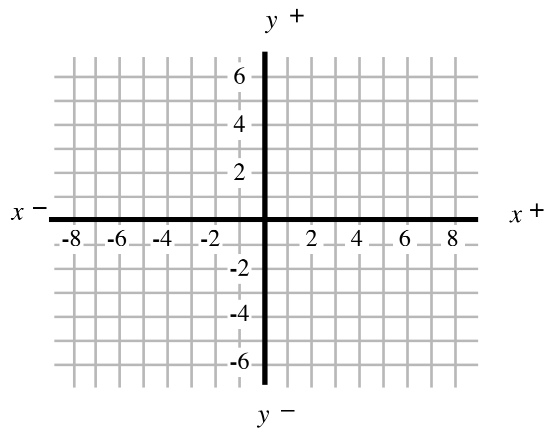


YTI 1 For each set of three non-collinear points, draw the three lines that pass through each pair. Use Example 1 as a guide.

a) (-2, 2), (3, 4), and (4, -2)



b) (-3, 0), (0, -5), and (2, 2)



LINEAR EQUATIONS OF THE FORM $ax + by = c$

In Section 4.1 you were introduced to the equation $x - y = 4$. This is a member of a family of linear equations of the form $ax + by = c$, where a, b, and c are numbers and x and y are variables.

> Linear equations with two variables, x and y, can be written in this form:
>
> $$ax + by = c$$
>
> Here, ***a***, ***b***, and ***c*** are numbers and ***x*** and ***y*** are variables.

~Instructor Insight~
The standard form of an equation is discussed in detail in Section 4.6. The equation $Ax + By = C$ will be referred to as the *standard form equation* at that time.

~Instructor Insight~
The graphing of horizontal and vertical lines is not discussed in this section. Instead, they are discussed in Section 4.6.

We can graph any line that has this form by finding three ordered pair solutions, and we often have a choice as to which three points are to be plotted. However, not all points on a line are easy to graph. For example, a point such as (21, 17) would not normally fit in our *x*-*y*-plane.

We can find the coordinates of a point by either:

1. choosing a value of *x*, placing it into the equation, and then solving for *y*, or

2. choosing a value of *y*, placing it into the equation, and then solving for *x*.

Often, one point can be found by choosing $x = 0$, and another by choosing $y = 0$. Then, by choosing one other value of either *x* or *y*, we will have three points to plot and can draw the line associated with the linear equation.

~Instructor Insight~

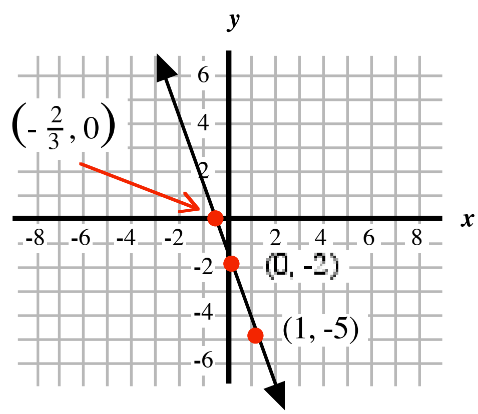
The *x*- and *y*-intercepts are not introduced here. They are discussed in detail in Sections 4.3 and 4.6.

Example 2: Graph the line of the equation $3x + y = -2$ by first finding three ordered pair solutions.

Procedure: For this example, let's choose $x = 0$, $y = 0$, and $x = 1$. We'll set up a table to keep our work organized. (Be careful to place the *x*- and *y*-coordinates correctly in the ordered pair.)

Answer:

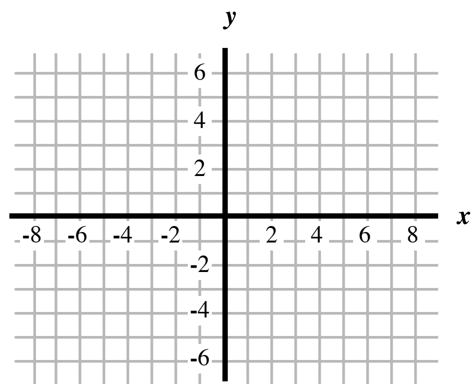
Choose a value	$3x + y = -2$	(x, y)
$x = 0$	$3(0) + y = -2$ $0 + y = -2$ $y = -2$	$(0, -2)$
$y = 0$	$3x + 0 = -2$ $3x = -2$ $\frac{3x}{3} = \frac{-2}{3}$ $x = -\frac{2}{3}$	$\left(-\frac{2}{3}, 0\right)$
$x = 1$	$3(1) + y = -2$ $3 + y = -2$ $3 + (-3) + y = -2 + (-3)$ $y = -5$	$(1, -5)$



Caution:	If the three points we plot are not on the same line, then we made a mistake somewhere, and we must correct it before we can graph the line.

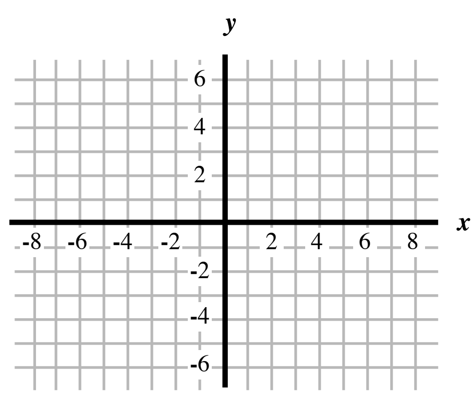
YTI 2 Graph the line of the equation $2x - y = 4$ by first finding three sets of ordered pair solutions. Use these values: $x = 0$, $y = 0$, and $y = 2$. Use Example 2 as a guide.

Choose a value	$\mathbf{2x - y = 4}$	(x, y)
$x = 0$		
$y = 0$		
$y = 2$		



YTI 3 Graph the line of the equation $x + 2y = 6$ by first finding three sets of ordered pair solutions. You choose the three values to use. Use Example 2 as a guide.

Choose a value	$\mathbf{x + 2y = 6}$	(x, y)



LINEAR EQUATIONS OF THE FORM $y = mx + b$

Another form of a linear equation is $y = mx + b$. Written in this form, we say that y *is in terms of* x. Because y is already isolated on the left side, we can choose three values of x to find the three points.

> Linear equations written with y in terms of x have the form
>
> $$y = mx + b$$
>
> Here, m and b are numbers and x and y are variables.

~Instructor Insight

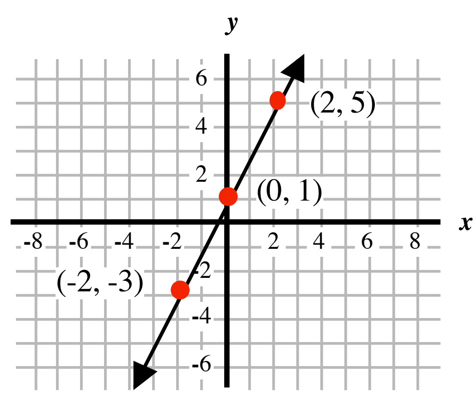
The slope and the y-intercept are discussed in detail in Section 4.3. The equation $y = mx + b$ will be referred to then as the *slope-intercept form*.

Example 3: Graph the line of the equation $y = 2x + 1$ by first finding three ordered pair solutions.

Procedure: For this example, let's choose $x = 0, 2$, and -2. We'll set up a table to keep our work organized.

Answer:

x	$y = 2x + 1$	(x, y)
0	$y = 2(0) + 1$ $y = 0 + 1$ $y = 1$	(0, 1)
2	$y = 2(2) + 1$ $y = 4 + 1$ $y = 5$	(2, 5)
- 2	$y = 2(-2) + 1$ $y = -4 + 1$ $y = -3$	(-2, -3)



We sometimes must be careful with the values of x we choose.

For example, in the equation $y = 2x + 1$, choosing $x = 10$ gives $y = 21$. The ordered pair (10, 21) is a solution, and there is such a point that can be plotted, but this would not fit in a typical x-y-plane.

Instead, choose relatively small values of x, somewhat close to 0. In fact, a good value to choose for x is 0 itself.

YTI 4 Graph the line of each given equation by first finding three sets of ordered pairs solutions. Use Example 3 as a guide.

a) $y = -2x + 3$

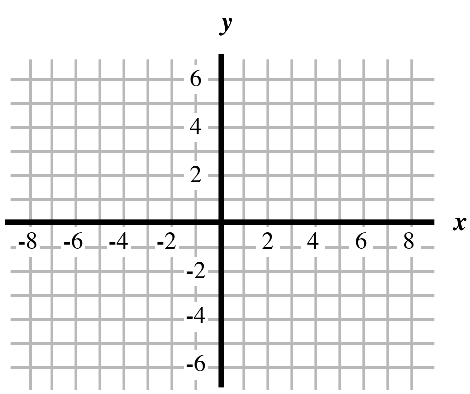
b) $y = 3x - 1$

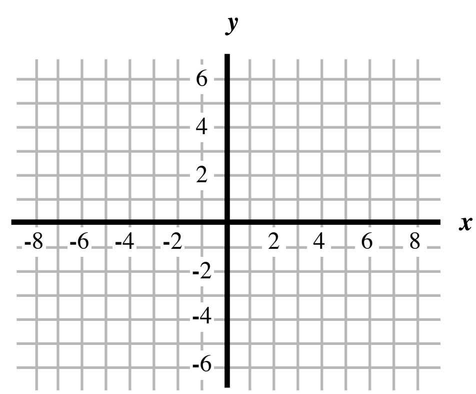
a)

x	$y = -2x + 3$	(x, y)

b)

x	$y = 3x - 1$	(x, y)





Think about it 1	Consider the equation $y = \frac{1}{2}x - 5$. What values of x would you choose to find *four* points on the line? Explain your answer.

____

____

A FRACTIONAL COEFFICIENT OF *x*.

Some linear equations of the form $y = mx + b$ have a fractional coefficient of x. In such cases, it's best to choose values of x that are multiples of the denominator. Doing so will typically result in integer values for y.

For example, consider the equation $y = \frac{2}{3}x + 2$. Because the denominator is 3, we should choose values of x that are multiples of 3, such as +3, -3, +6, -6 and 0.

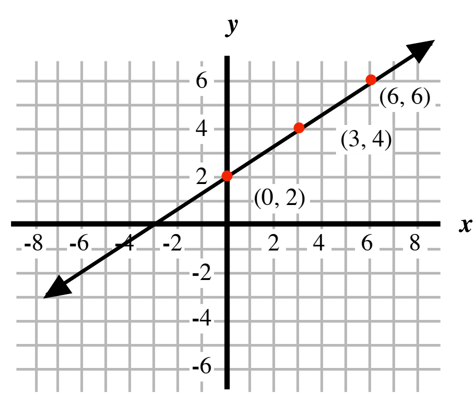
As has been mentioned, we need choose only three of these x-values, so we might choose 0, 3, and 6, or maybe -3, 0, and 3. Let's see.

Example 4: For the equation $y = \frac{2}{3}x + 2$, find three sets of ordered pairs as points in the x-y plane. Draw the line that passes through these points.

Procedure: Choose three values of x and find the corresponding y values. In this case, choose values of x that are multiples of 3.

Answer:

x	$y = \frac{2}{3}x + 2$	(x, y)
0	$y = \frac{2}{3} \cdot 0 + 2$ $y = 0 + 2$ $y = 2$	(0, 2)
3	$y = \frac{2}{3} \cdot 3 + 2$ $y = 2 + 2$ $y = 4$	(3, 4)
6	$y = \frac{2}{3} \cdot 6 + 2$ $y = 4 + 2$ $y = 6$	(6, 6)



Notice the graph (and the equation) in Example 8. We can see that there are two more points on the line that we can easily identify: (-3, 0) and (-6, -2). Each of these points has an *x*-values that is a multiple of 3.

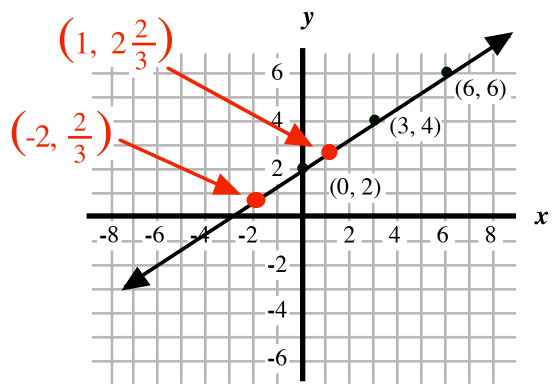
What would happen if we had chosen a value of *x* that is not a multiple of 3? Would we still get a point on the line? Let's explore this question by choosing $x = 1$ and $x = -2$:

x	$y = \frac{2}{3}x + 2$	(x, y)
1	$y = \frac{2}{3}(1) + 2$ $y = \frac{2}{3} + 2$ $y = \frac{2}{3} + \frac{6}{3}$ $y = \frac{8}{3}$ $y = 2\frac{2}{3}$	$\left(1, 2\frac{2}{3}\right)$

x	$y = \frac{2}{3}x + 2$	(x, y)
-2	$y = \frac{2}{3}(-2) + 2$ $y = \frac{-4}{3} + 2$ $y = \frac{-4}{3} + \frac{6}{3}$ $y = \frac{2}{3}$ $y = \frac{2}{3}$	$\left(-2, \frac{2}{3}\right)$

Both points $\left(1, 2\frac{2}{3}\right)$ and $\left(-2, \frac{2}{3}\right)$ are on the line, as we can see on the graph.

These points are a bit more challenging to plot in the *x*-*y*-plane because of the fractions in the *y*-coordinates.



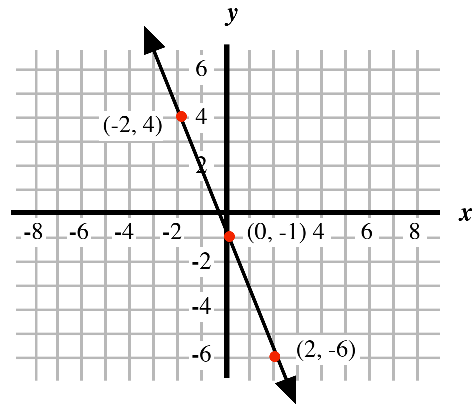
So, we can choose any value of *x* and find the corresponding value of *y*, but it is best to choose values of *x* that are multiples of the denominator.

Example 5: For the equation $y = \frac{-5}{2}x - 1$, find three sets of ordered pairs as points in the *x*-*y* plane. Draw the line that passes through these points.

Procedure: Choose three values of *x* and find the corresponding *y* values. In this case, choose values of *x* that are multiples of 2.

Answer:

x	$y = \frac{-5}{2}x - 1$	(x, y)
0	$y = \frac{-5}{2} \cdot 0 - 1$ $y = 0 - 1$ $y = -1$	(0, -1)
2	$y = \frac{-5}{2} \cdot 2 - 1$ $y = -5 - 1$ $y = -6$	(2, -6)
-2	$y = \frac{-5}{2} \cdot (-2) - 1$ $y = 5 - 1$ $y = 4$	(-2, 4)



YTI 5 For each equation, find three sets of ordered pairs as points in the *x*-*y* plane. Draw the line that passes through these points. Use Examples 4 and 5 as guides.

a) $y = \frac{3}{5}x - 3$

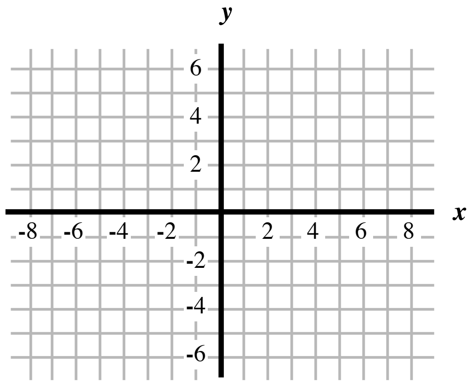
b) $y = \frac{-1}{4}x + 5$

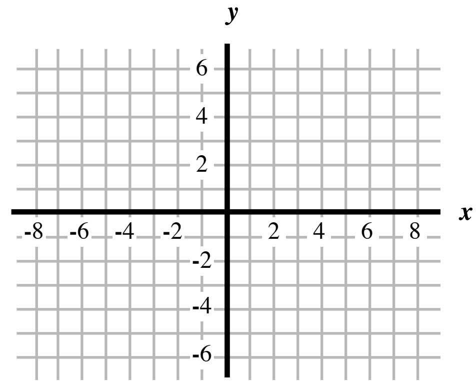
a)

x	$y = \frac{3}{5}x - 3$	(x, y)

b)

x	$y = \frac{-1}{4}x + 5$	(x, y)





YTI 6 For each equation, find three sets of ordered pairs as points in the *x*-*y* plane. Draw the line that passes through these points. Use Examples 4 and 5 as guides.

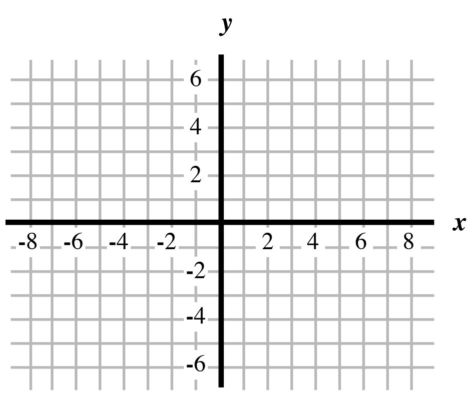
a) $y = -\frac{1}{2}x + 4$ b) $y = \frac{3}{2}x - 4$

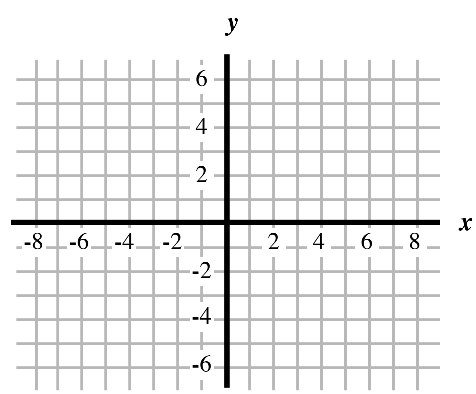
a)

x	$y = -\frac{1}{2}x + 4$	(x, y)

b)

x	$y = \frac{3}{2}x - 4$	(x, y)





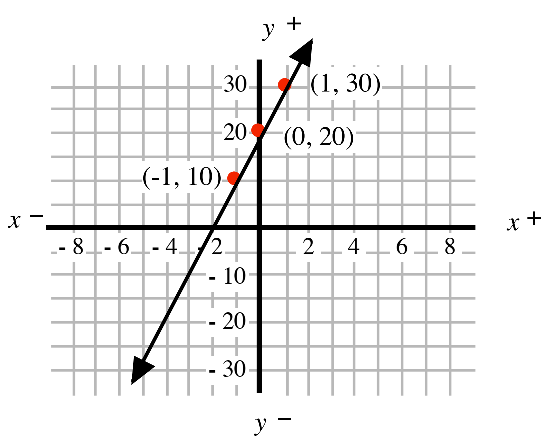
GRAPHING LINES WITH A LARGE SCALE

Sometimes a graph will have either x- or y-coordinates that do not fit easily on our typical x-y-plane. Consider, for example, the equation $y = 10x + 20$. If we choose values of x such as -1, 0, and 1, as shown at right, then all of the points found will be above or below our typical graph.

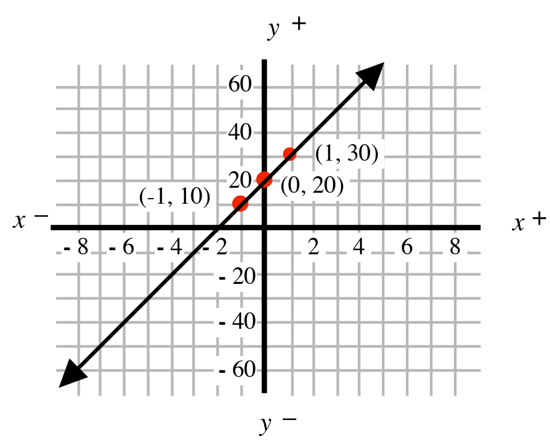
x	$y = 10x + 20$	(x, y)
- 1	$y = 10(-1) + 20$ $y = -10 + 20$ $y = 10$	(- 1, 10)
0	$y = 10(0) + 20$ $y = 0 + 20$ $y = 20$	(0, 20)
1	$y = 10(1) + 20$ $y = 10 + 20$ $y = 30$	(1, 30)

We can still graph the line, but we must create an x-y-plane that has a larger scale on the y-axis. To create a larger scale, we make each grid line 5 or 10 (or more) times the normal y-value. For this example, let's see it done with two different scales.

5 times the normal *y*-value scale



10 times the normal *y*-value scale



These two lines represent the same line, $y = 10x + 20$, even though they appear to have a different slant to them. The different slant is due to the different scales being used.

YTI 7 Graph the line of the given equation by first finding three ordered pairs solutions. Use the values of x given in the table.

a) $y = 10x - 5$

b) $y = -40x + 20$

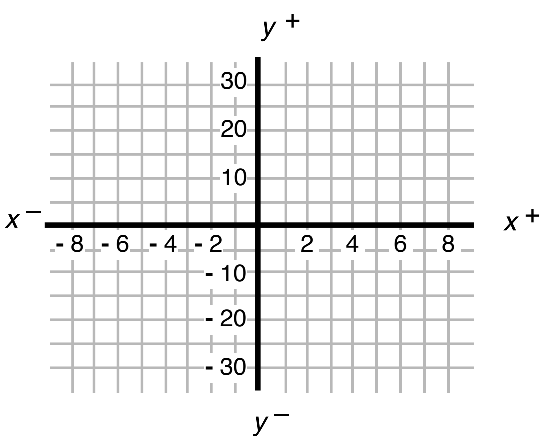
a)

x	$y = 10x - 5$	(x, y)
- 2		
0		
2		
3		

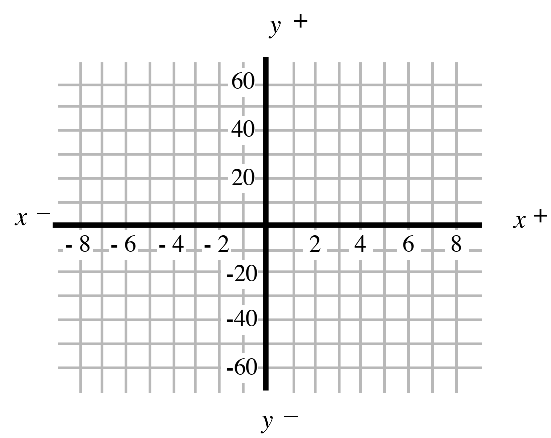
b)

x	$y = -40x + 20$	(x, y)
-1		
0		
1		
2		

5 times normal *y*-value scale



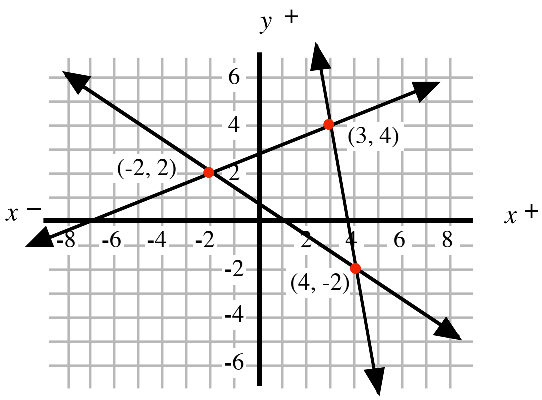
10 times normal *y*-value scale



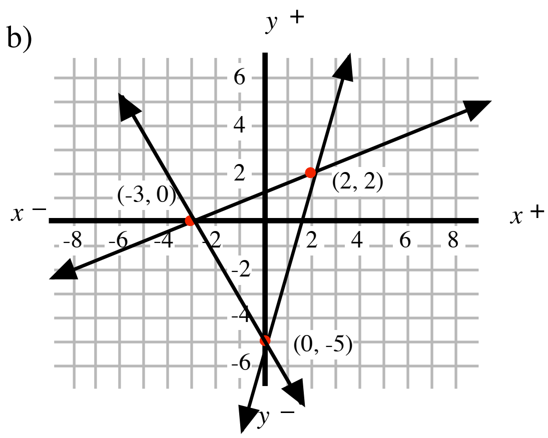
Answers: You Try It and Think About It

Some of the points you find may be different from the ones displayed in this answer set, but they should be on the same line.

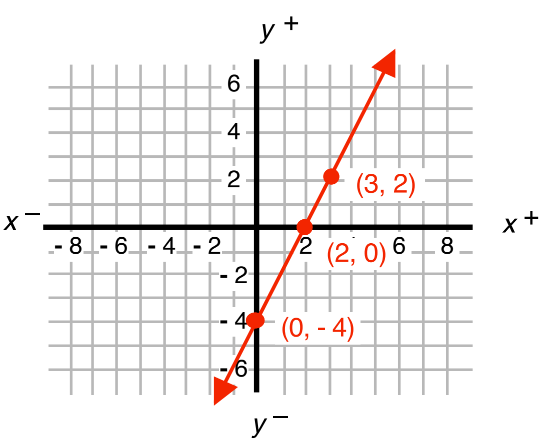
YTI 1:



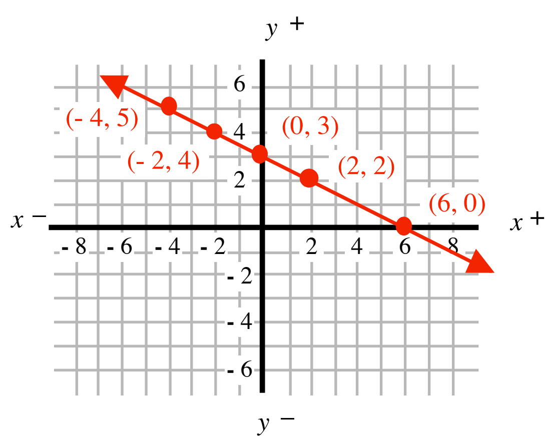
b)



YTI 2:

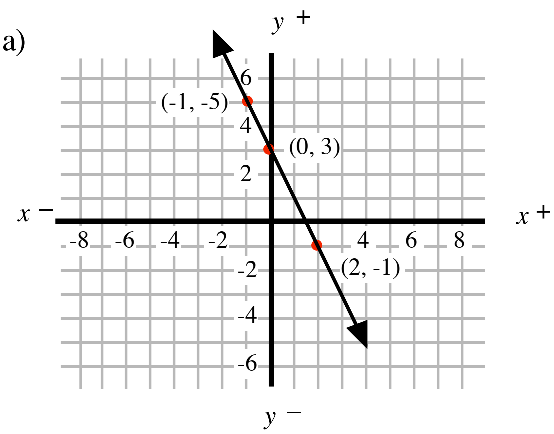


YTI 3: Some points shown here may be different from yours.

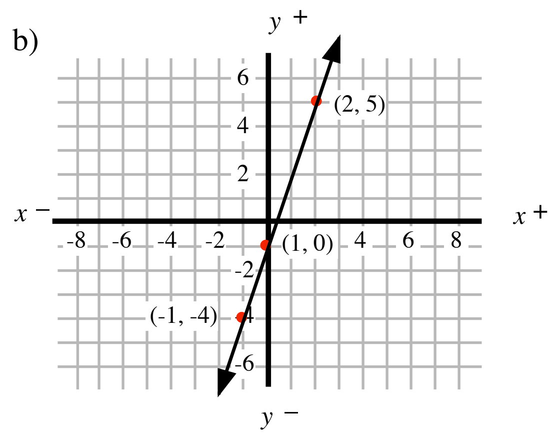


YTI 4: Some points shown here may be different from yours.

a)

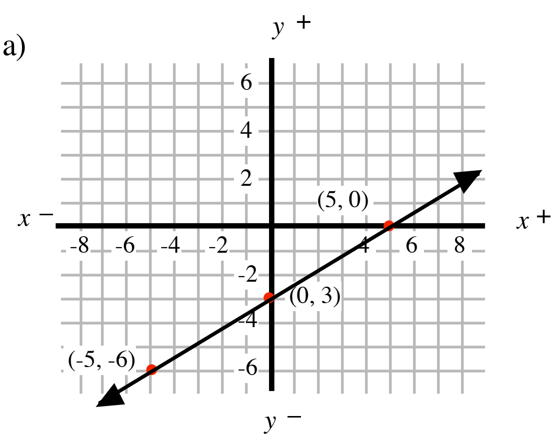


b)

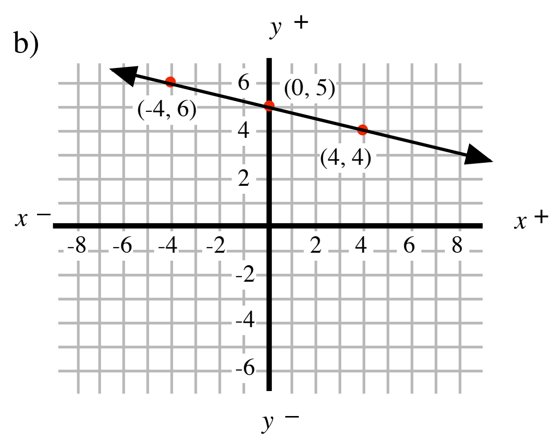


YTI 5: Some points shown here may be different from yours.

a)

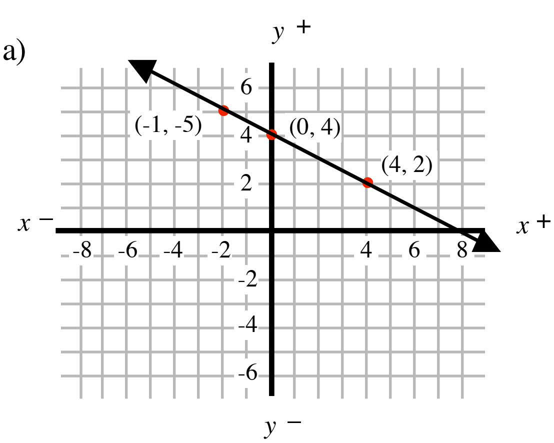


b)

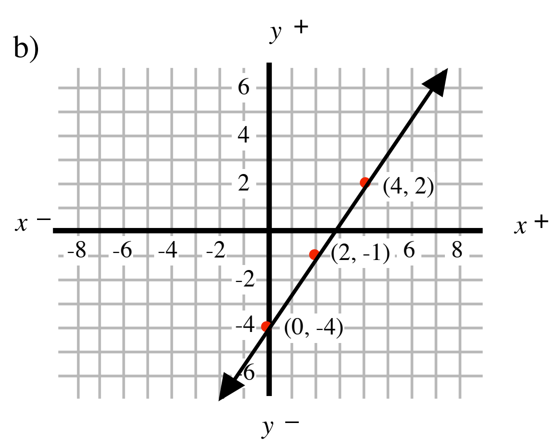


YTI 6: Some points shown here may be different from yours.

a)

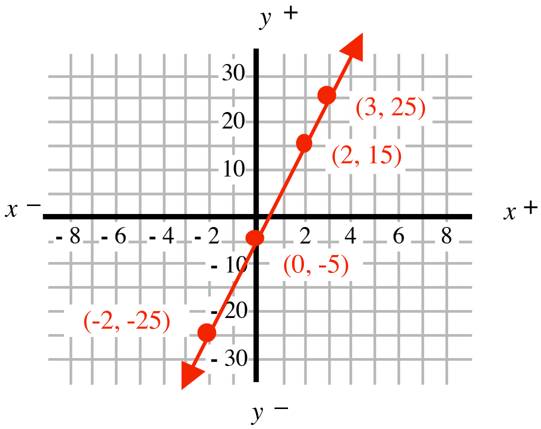


b)

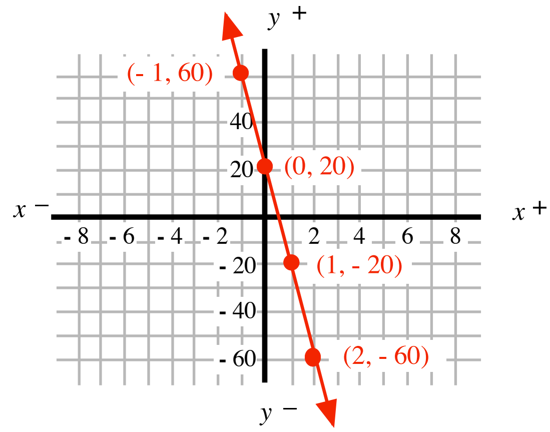


YTI 7:

a) 5 times the normal *y*-value scale



b) 10 times the normal *y*-value scale



Think About It: **1.** Answers may vary. One possibility is, Good choices for *x* are multiples of 2, such as 0, 2, -2, 4, and -4. Each of these values, when placed in the equation for *x*, help to eliminate the fraction in the equation.

Section 4.2 Exercises

Think Again.

1. Consider the equation $y = \frac{2}{3}x + 5$. What values of *x* would you choose to find *four* points on the line? Explain your answer. *(Think about it #1)*

2. What is another name for the graph of $y = 0$?

3. Three points in the *x*-*y*-plane are not on the same line. How many lines can be drawn in the plane that pass through at least two of the points? Explain your answer or show an example that supports your answer. You may want to draw a graph to assist you in your explanation.

4. Why is it good to find at least three points on a line before graphing the line?

Focus Exercises.

Graph the line with the given equation by first finding three points on the line.

5. $x + y = 6$ **6.** $x + y = -1$ **7.** $x - y = -3$ **8.** $x - y = 2$

9. $x + y = 0$ **10.** $x - y = 0$ **11.** $x + 2y = 4$ **12.** $x + 3y = 6$

13. $x - 2y = -2$ **14.** $x - 3y = -3$ **15.** $2x + y = -2$ **16.** $3x + y = -3$

17. $4x + y = 4$ **18.** $2x + y = -2$ **19.** $4x - y = 0$ **20.** $5x - y = 0$

Graph the line with the given equation by first finding three points on the line.

21. $y = x + 3$ **22.** $y = x + 5$ **23.** $y = x - 2$ **24.** $y = x - 6$

25. $y = -x + 4$ **26.** $y = -x + 1$ **27.** $y = -x - 3$ **28.** $y = -x$

29. $y = 2x - 3$ **30.** $y = 3x - 1$ **31.** $y = -3x + 4$ **32.** $y = -2x + 3$

33. $y = 4x$ **34.** $y = 5x$ **35.** $y = -2x$ **36.** $y = -3x$

37. $y = \frac{1}{2}x - 5$ **38.** $y = \frac{1}{3}x - 4$ **39.** $y = -\frac{1}{2}x + 2$ **40.** $y = -\frac{1}{3}x + 5$

41. $y = \frac{3}{2}x - 4$ **42.** $y = \frac{2}{3}x + 3$ **43.** $y = \frac{1}{4}x - 3$ **44.** $y = -\frac{3}{4}x + 1$

Graph the line with the given equation by first finding three points on the line. For each, a suggested larger scale is given.

45. $y = 10x - 15$

Use a scale that is 5 times the normal scale.

46. $y = 30x - 20$

Use a scale that is 10 times the normal scale.

47. $y = -40x - 20$

Use a scale that is 20 times the normal scale.

48. $y = -150x + 50$

Use a scale that is 50 times the normal scale.

Think Outside the Box.

Plot each pair of lines (Line A and Line B) in the same *x*-*y*-plane. Identify the point where they cross each other. Verify that the point found is a solution of each equation.

49. **Line A:** $y = 2x - 2$

Line B: $y = -\frac{2}{3}x + 6$

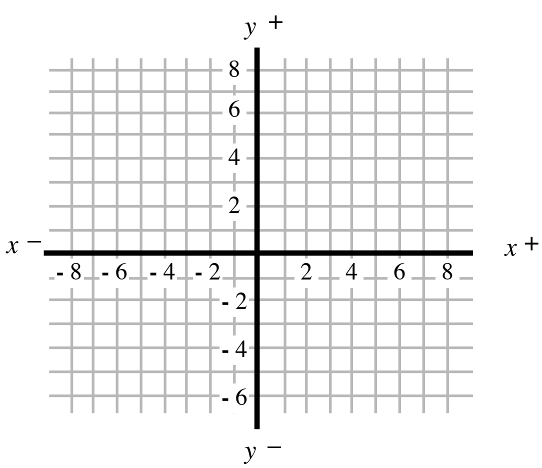
50. **Line A:** $y = -\frac{1}{3}x - 4$

Line B: $y = \frac{4}{3}x + 6$

*The graph of the equations, below, are not lines. Instead the points are connected by smooth curves. The graphs are called **parabolas**. For each, find the ordered pairs and plot them in the x-y-plane. Then connect the dots as smoothly as possible to form a parabola.*

51. $y = x^2 - 4x + 3$

x	$y = x^2 - 4x + 3$	(x, y)
-1		
0		
1		
2		
3		
4		
5		



52. $y = x^2 + 2x - 3$

x	$y = x^2 + 2x - 3$	(x, y)
-4		
-3		
-2		
-1		
0		
1		
2