Section 4.4 The Slope Formula

Objectives
In this section, you will learn to:

- Find the slope of a line through two points by counting rise and run.
- Create the equation of a graphed line.
- Find the slope of a line using the slope formula.
- Create the equation of a line using the slope formula.

To successfully complete this section, you need to understand:

- Placing values into formulas (1.8)
- Plotting points in the *x*-*y*-plane (4.1)
- The *y*-intercept point (4.3)
- The slope of a line (4.3)

INTRODUCTION

In Section 4.3, from a single point, we used the slope of the line to locate other points on the line. We begin this section by seeing points already on a line and using the idea of rise and run to find the slope of the line. We must keep three things in mind when counting rise and run:

1. The rise: counting upward is a positive rise and downward is a negative rise.

2. The run: counting to the right is a positive run, and to the left is a negative run.

3. The slope is always *rise over run*, $m = \frac{\text{rise}}{\text{run}}$, so count the rise first and then the run.

THE SLOPE OF A LINE FROM A GRAPH

If we know two points on the graph of a line, we can find the slope of the line by starting at one point and counting spaces toward the other. We first count the rise—up or down—and then the run—left or right.

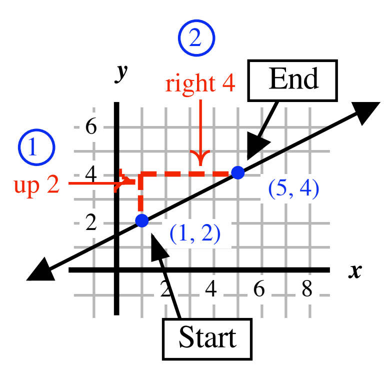
For example, if a line contains the points (1, 2) and (5, 4), then we can either start at (1, 2) and count toward (5, 4) or we can start at (5, 4) and count toward (1, 2). In either case the slope is the same.

A. Start at (1, 2)

1. The rise is up 2

2. The run is right 4

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{up } 2}{\text{right } 4} = \frac{+2}{+4} = \frac{1}{2}$$

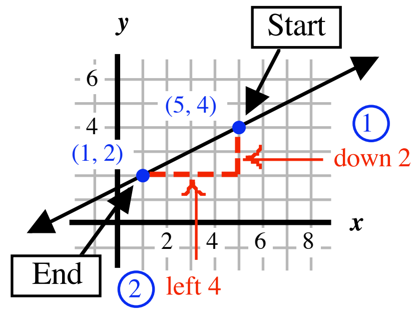


B. Start at (5, 4)

1. The rise is down 2

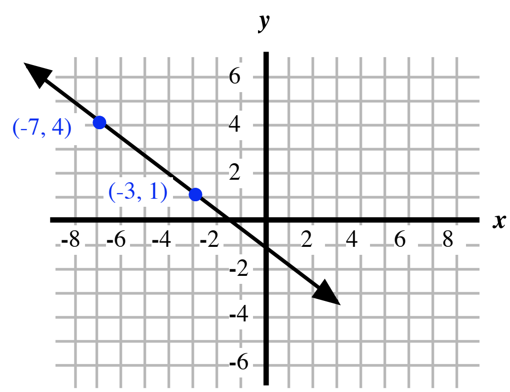
2. The run is left 4

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{down } 2}{\text{left } 4} = \frac{-2}{-4} = \frac{1}{2}$$



Example 1: Given the graph of the line, find the slope by counting the rise and the run.

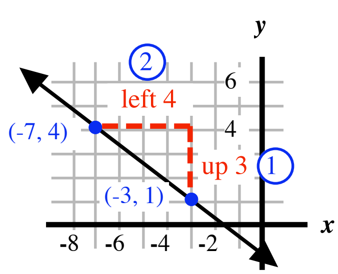
Procedure: Start at either point. Count the rise first and then the run. Then write the slope as $\frac{\text{rise}}{\text{run}}$ and simplify.



Answer: (Two answers are shown for this example, but only one is necessary.)

One option:

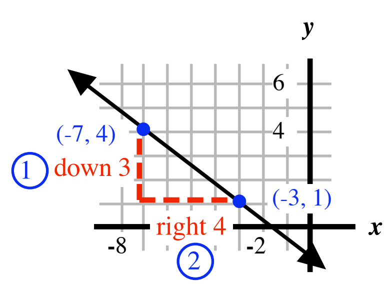
Start at (-3, 1):



$$m = \frac{\text{up } 3}{\text{left } 4} = \frac{+3}{-4} = -\frac{3}{4}$$

A second option:

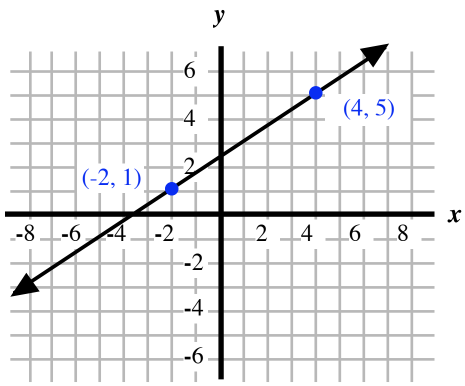
Start at (-7, 4):



$$m = \frac{\text{down } 3}{\text{right } 4} = \frac{-3}{+4} = -\frac{3}{4}$$

YTI 1 Given the graph of the line, find the slope by counting the rise and the run. Use Example 1 as a guide.

a)



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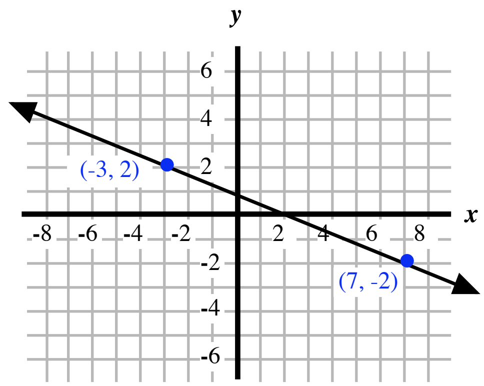


GRAPHING A LINE FROM ITS INTERCEPTS

If we know both the *x*- and *y*-intercept points, then we can plot these points and find the slope by counting the rise and run. We can then use the slope to find one or two more points on the line and graph the line.

Example 2: Given the *x*- and *y*-intercept points, plot them and identify the slope. Use the slope to find two more points on the line and graph the line.

a) (3, 0) and (0, -2)

b) (-4, 0) and (0, -3)

Procedure: Plot the points and count the rise and run to identify the slope.

a) Counting from the *y*-intercept point, the rise is *up 2* and the run is *right 3*.

The slope is $\frac{+2}{+3}$, and we can use

$m = \frac{2}{3} = \frac{\text{up 2}}{\text{right 3}}$ and $m = \frac{-2}{-3} = \frac{\text{down 2}}{\text{left 3}}$

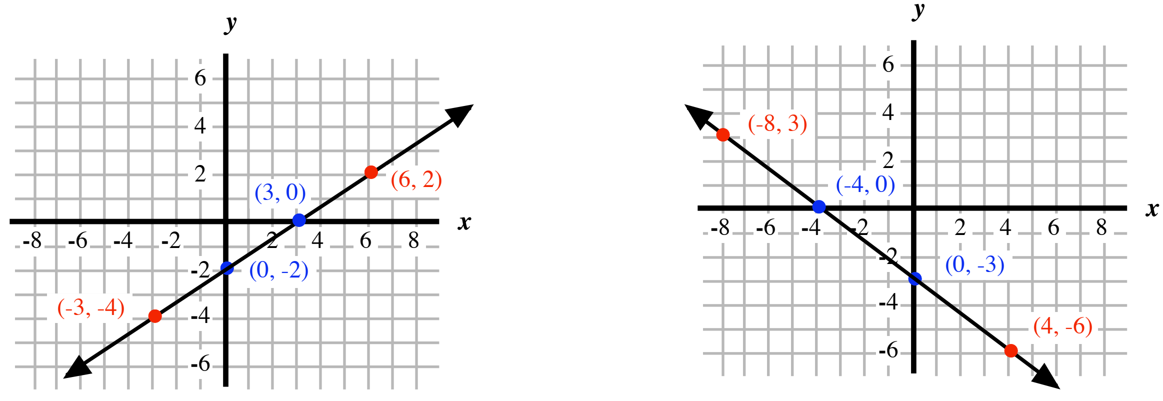
to locate other points on the line.

b) Counting from the *y*-intercept point, the rise is *up 3* and the run is *left 4*.

The slope is $\frac{+3}{-4}$, and we can use

$m = \frac{3}{-4} = \frac{\text{up 3}}{\text{left 4}}$ and $m = \frac{-3}{4} = \frac{\text{down 3}}{\text{right 4}}$

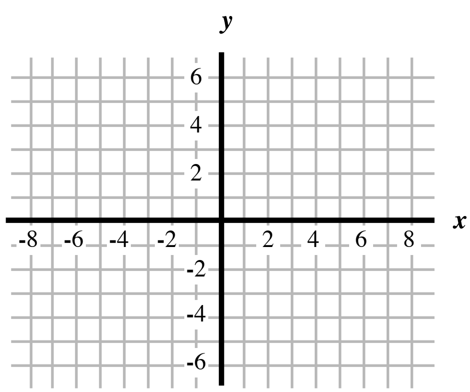
to locate other points on the line.

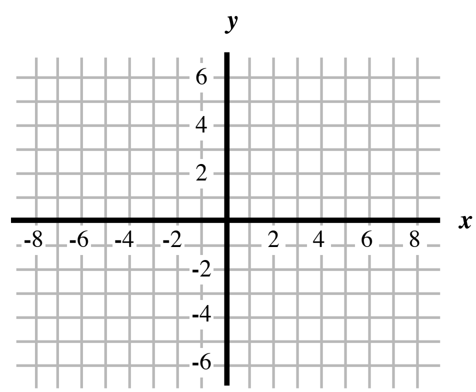


YTI 2 Given the x- and y-intercept points, plot them and identify the slope. Use the slope to find two more points on the line and graph the line. Use Example 2 as a guide.

a) (1, 0) and (0, 3)

b) (-4, 0) and (0, 2)



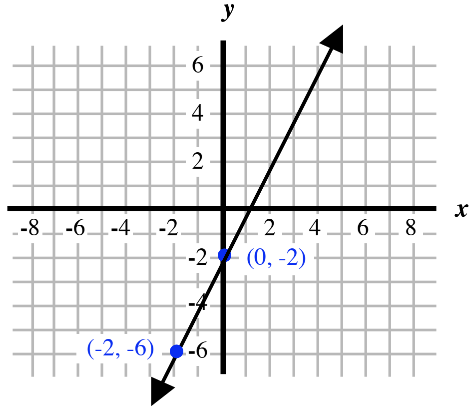


Creating the Equation for the Graph

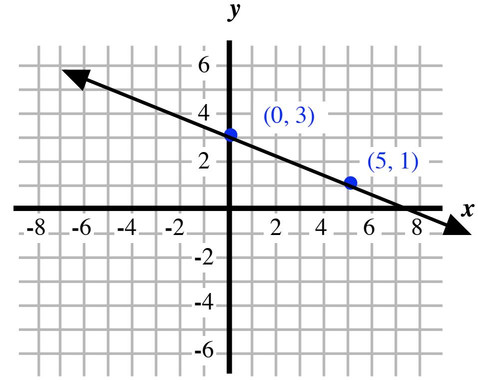
For most lines, if we can see the graph of a line, then we can write the equation that goes with that line. The key is identifying the y-intercept $(0, b)$ and the slope, m; then we use the form $y = mx + b$ to write the equation.

Example 3: Given the graph of the line, identify its slope and y-intercept, and use them to write the equation of the line.

a)



b)



Procedure: Identify the y-intercept point, $(0, b)$, giving us the value of b. Next, identify the slope by counting the rise and run, starting with the y-intercept point; simplify m. Write the equation of the line using the slope-intercept form $y = mx + b$.

a) The y-intercept point is (0, -2), so $b = -2$.

Counting from (0, -2) to (-2, -6), we see

a rise of -4 and a run of -2, so

$$m = \frac{-4}{-2} = 2.$$

The equation of the line is

Answer: $y = 2x - 2$

b) The y-intercept point is (0, 3), so $b = 3$.

Counting from (0, 3) to (5, 1) we see

a rise of -2 and a run of 5, so

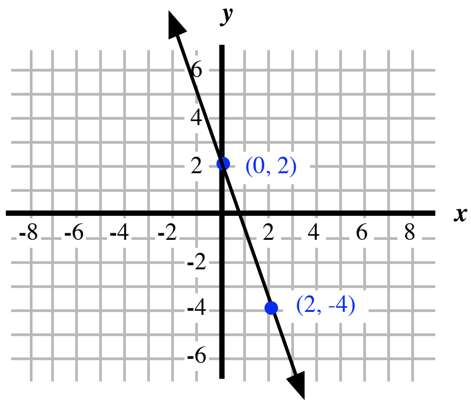
$$m = \frac{-2}{5} = -\frac{2}{5}.$$

The equation of the line is

$$y = -\frac{2}{5}x + 3$$

YTI 3 Given the graph of the line, identify its slope and y-intercept, and use them to write the equation of the line. Use Example 3 as a guide.

a)



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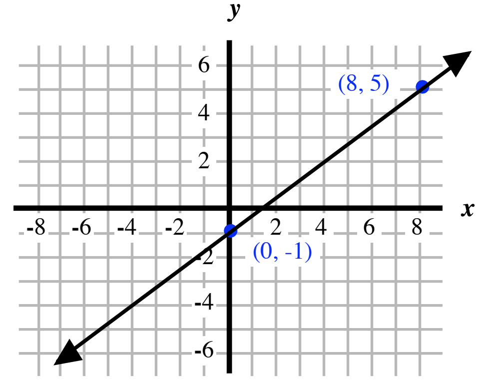

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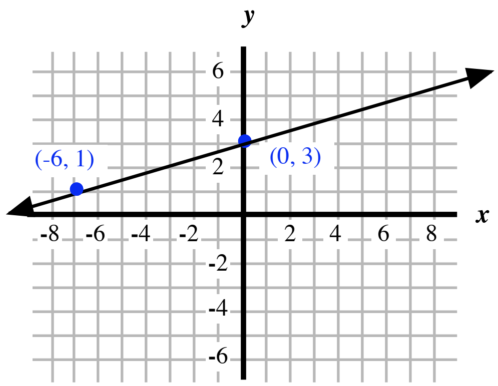
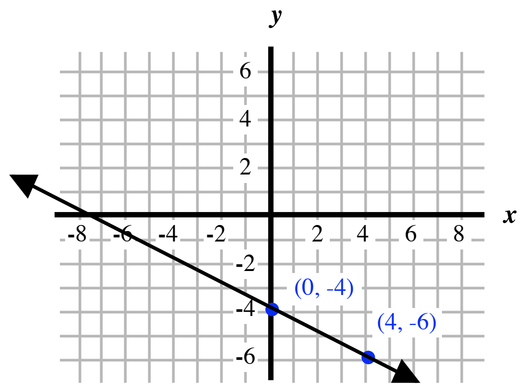


d)



THE SLOPE FORMULA

Recall from Section 4.3 the distinction between *distance* and *difference*. Both are found using subtraction, but distance is always a positive measure, whereas difference can be positive or negative. Consider this example:

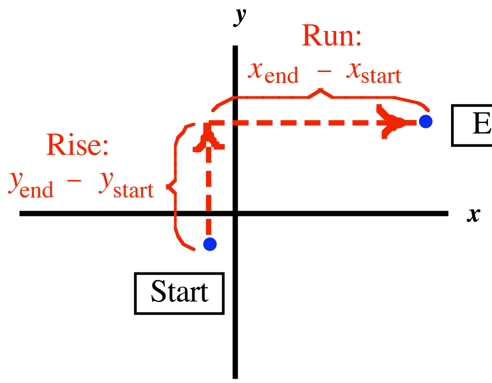
> Dayna drove from her home in California to her college in Missouri. At the start of the trip, the car's odometer (mileage indicator) read 030253 and at the end of the trip it read
> 031877 . How far did Dayna drive on this trip? We calculate the distance by subtraction:

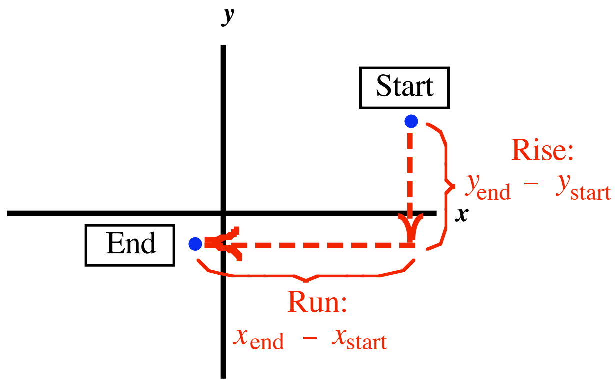
end mileage – *start* mileage = 31,877 – 30,253 = 1,624 miles.

This idea of subtracting the starting value from the ending value can be extended to the rise and run differences in the slope of a line. The rise is the difference of y-values, $y_{end} - y_{start}$, and the run is the difference of x-values, $x_{end} - x_{start}$. This means that we can write the slope as

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_{end} - y_{start}}{x_{end} - x_{start}}$$

As these two diagrams demonstrate, either point can be the starting point.





However, instead of subscripts $_{start}$ and $_{end}$, we usually use $_1$ and $_2$ to represent the first point (x_1, y_1) and the second point (x_2, y_2). This leads us to the *slope formula*:

The Slope Formula:

The slope of the line that passes through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

For example, if we want to find the slope of the line that passes through the points (3, 2) and (7, 8), then we can designate either one to be the first point: We'll designate which is the first point and which is the second by placing (x_1, y_1) over one point and (x_2, y_2) over the other point.

Choosing $\overset{(x_1,\, y_1)}{(3,\ 2)}$ and $\overset{(x_2,\, y_2)}{(7,\ 8)}$ the slope formula is $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 2}{7 - 3} = \dfrac{6}{4} = \dfrac{3}{2}$.

Instead, choosing $\overset{(x_1,\, y_1)}{(7,\ 8)}$ and $\overset{(x_2,\, y_2)}{(3,\ 2)}$ the slope formula is $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2 - 8}{3 - 7} = \dfrac{-6}{-4} = \dfrac{3}{2}$.

Notice that it doesn't matter which point we choose as the first point, the slope is the same.

Example 4: Find the slope of the line that passes through the given points and simplify.

a) (-5, 4) and (-1, 6) b) (1, -5) and (-2, 4)

Procedure: Use the slope formula. It doesn't matter which point is chosen as the first point and which as the second, either choice will lead to the same slope.

a) Let's choose (-5, 4) as the *first* point and (-1, 6) as the *second* point:

$$m = \frac{6 - 4}{-1 - (-5)} = \frac{6 - 4}{-1 + 5} = \frac{2}{4} = \frac{1}{2}$$

b) Let's choose (-2, 4) as the *first* point and (1, -5) as the *second* point:

$$m = \frac{-5 - 4}{1 - (-2)} = \frac{-5 - 4}{1 + 2} = \frac{-9}{3} = -3$$

YTI 4 Find the slope of the line that passes through the given points and simplify. Use Example 4 as a guide.

a) (1, 4) and (5, 10)

b) (4, -2) and (-6, 8)

c) (-6, 3) and (0, -7)

d) (-2, 0) and (0, 6)

Think about it 1 Given two points, (6, -10) and (-20, 15) decide whether the slope formula is set up correctly or not. If not, state what the error is.

a) $m = \dfrac{6 - (-20)}{-10 - 15}$ ____

b) $m = \dfrac{-10 - 15}{6 - (-20)}$ ____

c) $m = \dfrac{15 - (-10)}{-20 - 6}$ ____

d) $m = \dfrac{15 - (-20)}{-10 - 6}$ ____

We generally use the slope formula when we can't count to get the slope, because either

1. the points are not already plotted in the *x*-*y*-plane, or
2. one or both points have coordinates that are outside of the grid region that we have been using.

Caution: It can be easy to misplace some of the numbers within the slope formula. One strategy that might be helpful is to align the ordered pairs, one above the other, as shown in the diagram. Then place $m = \underline{\quad\quad}$ below this alignment and prepare to correctly place the numbers in the formula.

The preparation	Placing the values correctly
(x, y)	(x, y)
(7, 8)	(7, 8)
(3, 2)	(3, 2)
$m = \underline{\quad\quad}$	$m = \dfrac{8 - 2}{7 - 3}$

YTI 5 Find the slope of the line that passes through the given points and simplify. Use Example 4 as a guide.

a) (0, 0) and (-3, 6)

b) (-16, 3) and (0, -5)

c) (-11, -10) and (3, 2)

d) (-13, -12) and (-17, -11)

CREATING THE EQUATION OF A LINE USING THE SLOPE FORMULA

As you saw earlier in this section, if we have the right information, we can easily write the equation of the line. In particular, if we know the *y*-intercept point, (0, *b*), of a line, and if we can find the slope, *m*, then we can use the slope-intercept form of a line, $y = mx + b$, to write the equation.

Example 5: Find the equation of the line that passes through the given points.

a) (0, 0) and (1, -5)

b) (3, 1) and (0, -3)

Procedure: Notice that the *y*-intercept point, (0, *b*), is one of the given points in the pair. This means that we have the value of *b*. We can use the slope formula to find *m*, and place these values into $y = mx + b$.

a) The *y*-intercept point is (0, 0), so $b = 0$.

Find the slope:

$$m = \frac{-5 - 0}{1 - 0} = \frac{-5}{1} = -5$$

The equation of the line is

Answer: $y = -5x + 0$, or $y = -5x$

b) The *y*-intercept point is (0, -3), so $b = -3$.

Find the slope:

$$m = \frac{-3 - 1}{0 - 3} = \frac{-4}{-3} = \frac{4}{3}$$

The equation of the line is

$$y = \frac{4}{3}x - 3$$

YTI 6 Find the equation of the line that passes through the given points. Use Example 5 as a guide.

a) (0, -5) and (4, 3)

b) (0, 8) and (12, 0)

c) (-4, 6) and (0, 4)

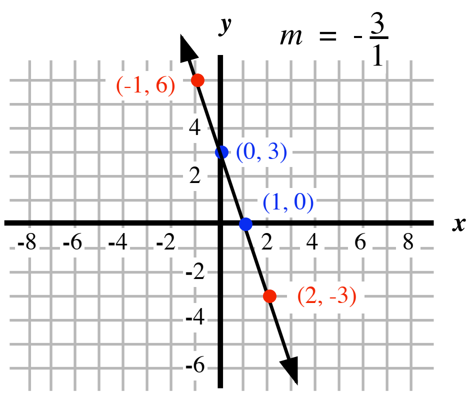
d) (0, 0) and (-4, -6)

Answers: You Try It and Think About It

YTI 1: a) $m = \frac{4}{6} = \frac{2}{3}$ b) $m = -\frac{4}{10} = -\frac{2}{5}$

YTI 2: Some points shown may be different from yours.

a)



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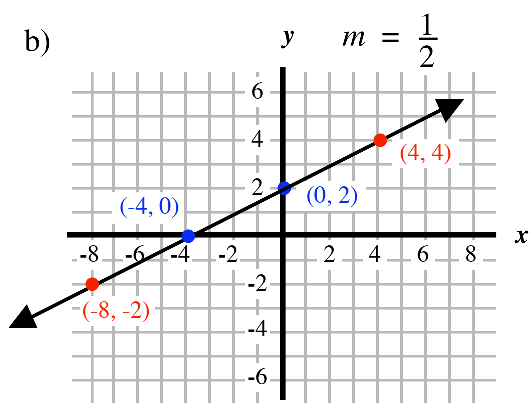


YTI 3: a) $y = -3x + 2$ b) $y = \frac{3}{4}x - 1$

c) $y = \frac{1}{3}x + 3$ d) $y = -\frac{1}{2}x - 4$

YTI 4: a) $m = \frac{3}{2}$ b) $m = -1$ c) $m = -\frac{5}{3}$ d) $m = 3$

YTI 5: a) $m = -2$ b) $m = -\frac{1}{2}$ c) $m = \frac{6}{7}$ d) $m = -\frac{1}{4}$

YTI 6: a) $y = 2x - 5$ b) $y = -\frac{2}{3}x + 8$

c) $y = -\frac{1}{2}x + 4$ d) $y = \frac{3}{2}x$

Think About It: **1.** a) is not correct; the difference in *x*-values is in the numerator. b) is correct. c) is correct. d) is not correct; both the numerator and denominator is a difference between an *x*-value and a *y*-value.

Section 4.4 Exercises

Think Again.

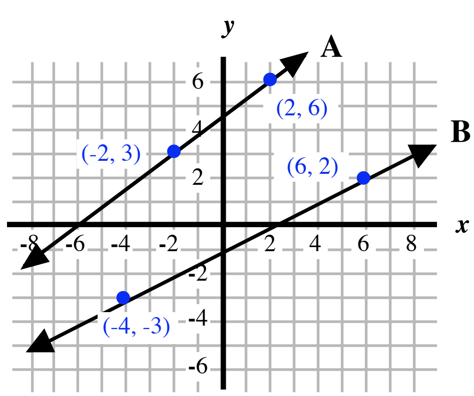
Given two points, (-8, -9) and (13, -18) decide whether the slope formula is set up correctly or not. If not, state what the error is. (Refer to Think About It 1)

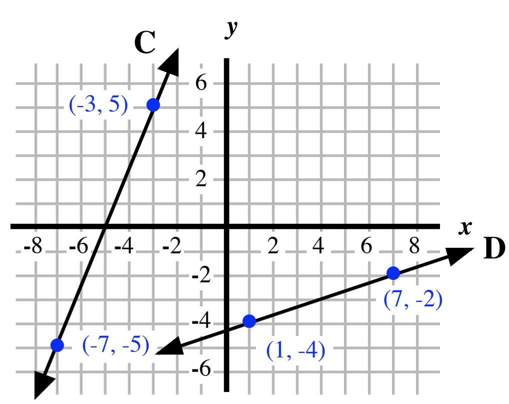
1. $m = \frac{-9 - (-18)}{-8 - 13}$ **2.** $m = \frac{-18 - 13}{-9 - (-8)}$ **3.** $m = \frac{-9 - (-18)}{13 - (-8)}$ **4.** $m = \frac{13 - (-8)}{-18 - (-9)}$

Focus Exercises.

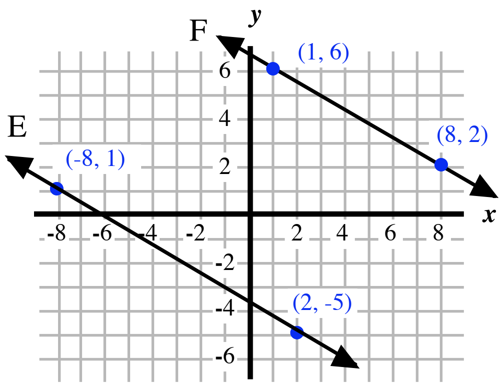
Given the graph of each line, find the slope by counting the rise and the run.

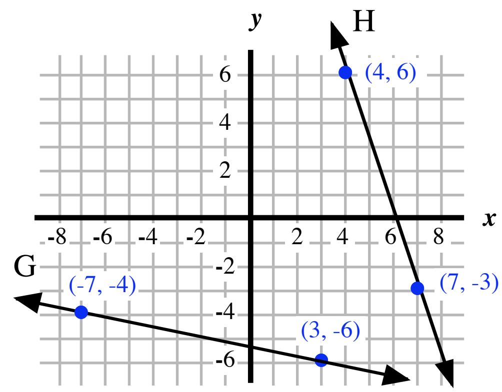
5. Line A **6.** Line B **7.** Line C **8.** Line D





9. Line E **10.** Line F **11.** Line G **12.** Line H





Given the x- and y-intercept points, plot them and identify the slope. Use the slope to find two more points on the line and graph the line.

13. (-3, 0) and (0, 2)

14. (6, 0) and (0, -2)

15. (0, 4) and (2, 0)

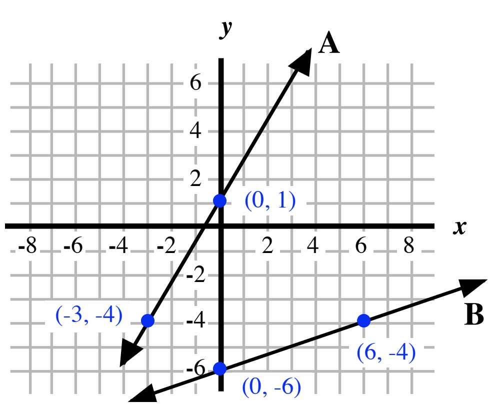
16. (0, -3) and (-1, 0)

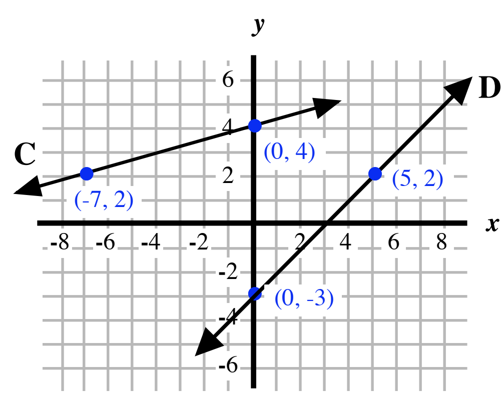
17. (-5, 0) and (0, 5)

18. (0, -6) and (6, 0)

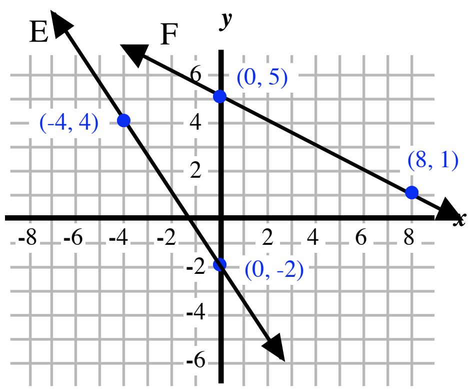
Given the graph of the line, identify its slope and *y*-intercept, and use them to write the equation of the line.

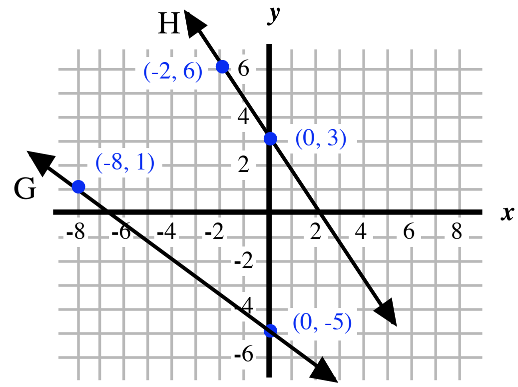
19. Line A **20.** Line B **21.** Line C **22.** Line D





23. Line E **24.** Line F **25.** Line G **26.** Line H





Use the slope formula to find the slope of the line that passes through the given points. Simplify, if possible.

27. (1, 2) and (4, 5)

28. (9, 8) and (6, 2)

29. (2, -1) and (6, 3)

30. (5, -4) and (2, 0)

31. (5, 3) and (9, -5)

32. (-2, 7) and (7, 10)

33. (0, -6) and (-4, 0)

34. (-4, 1) and (-7, -1)

35. (-3, -6) and (0, 0)

36. (9, -2) and (-3, -2)

37. (4, -5) and (4, 2)

38. (-1, -3) and (-5, -9)

39. (-6, 0) and (6, 3)

40. (-10, 5) and (6, -7)

41. (5, 7) and (0, 2)

42. (-5, 9) and (4, 0)

43. (-1, 5) and (4, 5)

44. (3, -2) and (1, -2)

45. (2, -7) and (2, 3)

46. (7, -4) and (7, -6)

Find the equation of the line that passes through the given points.

47. (0, 7) and (3, 16)

48. (0, -8) and (-2, -18)

49. (0, 11) and (14, 4)

50. (0, -6) and (15, 4)

51. (-9, 10) and (0, -8)

52. (-4, -10) and (0, -14)

53. (0, 0) and (9, -15)

54. (-12, 8) and (0, 0)

55. (0, 10) and (25, 0)

56. (24, 0) and (0, 16)

57. (-20, 0) and (0, -15)

58. (0, -18) and (-12, 0)

Think Outside the Box:

For each pair of points:

a) Plot the points in the x-y-plane.
b) Find the slope of the line.
c) Use the slope to locate the y-intercept.
d) Graph the line that passes through these two points
e) Write the equation of the line.

59. (6, 1) and (3, 2)

60. (8, 4) and (4, 1)

61. (-3, -6) and (-2, -4)

62. (-8, 3) and (-7, 2)

