Simplifying Square Roots

First, the square root of 25 can be written as either $\sqrt[2]{25}$ or just $\sqrt{25}$. The square root symbol $\sqrt{}$ is called the **radical** and the number within the radical is called the **radicand**.

The little **2** in $\sqrt[2]{25}$ is called the **index**. It indicates that we can simplify perfect squares.

Simplifying square roots requires

a) knowing how to factor numbers;

b) knowing what numbers are already perfect squares, like 4, 9, 16, 25; and

c) knowing what makes a perfect square: a number multiplied by itself.

So, 4 is perfect square because it is $2 \cdot 2$.

Likewise, 49 is perfect square because it is $7 \cdot 7$.

So, if we need to simplify, for example, $\sqrt{24}$ we can approach it one of two ways:

1) we can recognize that 24 has a lot of factors, such as 2, 3, **4**, 6, 8, and 12, but only one of them is a perfect square factor: 4

Using this method, we can rewrite $\sqrt{24}$ as $\sqrt{4 \cdot 6}$ and then separate those into $\sqrt{4} \cdot \sqrt{6}$. Once separated, we are able to simplify only the square root that has the perfect square within it: $\sqrt{4} = 2$

So, $\sqrt{4} \cdot \sqrt{6} = 2 \cdot \sqrt{6}$ and we're done.

2) Here is the second method, using prime factorization

Find the prime factorization of the number 24 and look for any factors that can be doubled up to create a perfect square:

$24 = \mathbf{2 \cdot 2} \cdot 2 \cdot 3$ even though there are *three* 2's, we need (and want) only two of them to make a perfect square: $2 \cdot 2 = 4$ The rest of the factors multiply to 6 so we again get 24 = 4 · 6:

$$\sqrt{24} = \sqrt{4 \cdot 6} = \sqrt{4} \cdot \sqrt{6} = 2 \cdot \sqrt{6} \quad \text{or} \quad 2\sqrt{6}$$

Sometimes the radicand is larger, such as $\sqrt{72}$. If we don't recognize a perfect square factor of 72, we should use prime factorization:

It turns out that 72 = **2 · 2** · 2 · **3 · 3**.
This time, there are double factors of 2 (**2 · 2** = 4) and double factors of 3 (**3 · 3** = 9).
This leads to 72 = **4** · **9** · 2 (2 is the only factor left over after the doubling up.)

So, $\sqrt{72}$

$$= \sqrt{4 \cdot 9 \cdot 2}$$

$$= \sqrt{4} \cdot \sqrt{9} \cdot \sqrt{2}$$

$$= 2 \cdot 3 \cdot \sqrt{2}$$

$$= 6 \cdot \sqrt{2} \quad \text{or just} \quad 6\sqrt{2}$$

Lastly, if a radical—before being simplified—is already multiplied by an integer, then that integer will need to be included in the final simplified form.

For example, Simplify $7\sqrt{50}$. First, 50 has a perfect square factor of 25, so we can rewrite this expression as $7\sqrt{25 \cdot 2}$. (Notice that the integer, 7, is still present as we simplify.)

Next, we can separate the radicand into two square roots:

$$7\sqrt{25 \cdot 2} = 7 \cdot \sqrt{25} \cdot \sqrt{2}$$

Only the $\sqrt{25}$ can be simplified to a whole number:

$$7\sqrt{25 \cdot 2} = 7 \cdot \sqrt{25} \cdot \sqrt{2} = 7 \cdot 5 \cdot \sqrt{2}$$

and the whole numbers can then be multiplied together:

$$7\sqrt{25 \cdot 2} = 7 \cdot \sqrt{25} \cdot \sqrt{2} = 7 \cdot 5 \cdot \sqrt{2} = 35 \cdot \sqrt{2}$$

I hope you find this helpful.

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